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LTM Cover Diagram, February 2006: A semi-regular side-gon

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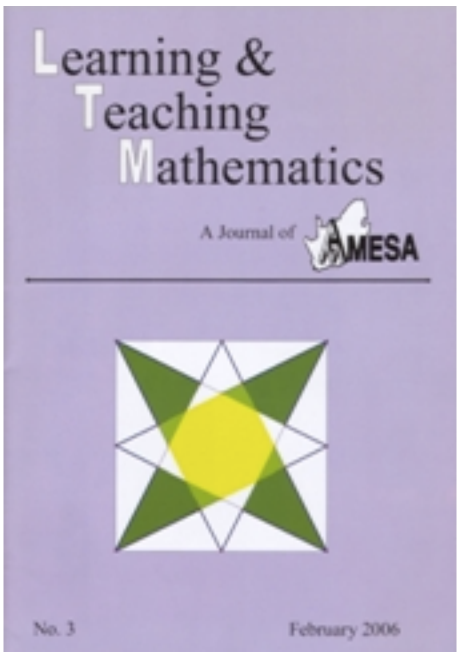
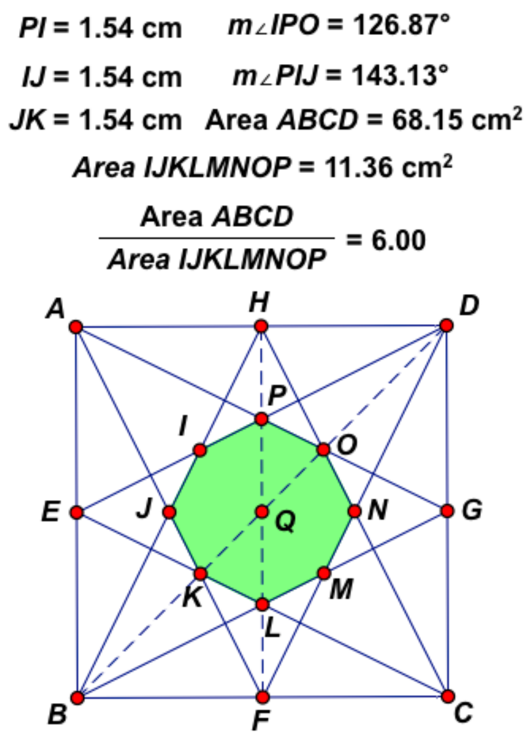
http://dynamicmathematicslearning.com/JavaGSPLinks.htm





The *Learning & Teaching Mathematics* journal, no. 3, February 2006 as shown in the first figure above had an appealing geometric design on the cover that begs some further exploration. At first glance, it may visually appear that the formed yellow octagon is regular, and be tempting to conjecture that it is regular (as many of my students do when asked).

However, this conjecture is false, since a regular octagon has rotational symmetry of order 8, but the original square from which it is constructed has rotational symmetry of only order 4: so the constructed octagon has rotational symmetry of only order 4, and hence cannot be regular. In fact, from this rotational symmetry it follows by repeated rotations of 90º of the second figure above that the alternate angles at *P*, *N*, *L* and *J* map onto each other and are equal, as are the alternate angles at *O*, *M*, *K* and *I*. The

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measurements of the angles at *P* and *I* are shown in the 2nd sketch above, but it is also a useful exercise in trigonometry to give students to determine the size of the two sets of equal alternate angles.

The octagon, however, has all its sides equal. This follows easily from the axes of symmetry of the square. For example, a reflection of *IP* around *HF* clearly maps it onto *OP*; hence *IP* = *OP*. Similarly, a reflection of *PI* around the diagonal *AC* of the square, gives us *PI* = *JI*. Continued reflections around the axes of symmetry show us that all the sides are equal - as illustrated by the measurement of 3 sides in the 2nd sketch above. This octagon is therefore an example of what I've called a *semi-regular side-gon* in De Villiers (2011), and is a generalization of the concept of a rhombus to hexagons, octagons, etc. An interactive *JavaSketchpad* sketch is available to explore some of the interesting properties of these (and other) polygons at: http://frink.machighway.com/~dynamicm/semi-regular-anglegon.html

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Another interesting property of this constructed, semi-regular octagon is that its area is 1/6 that of the square – as shown by the measurements in the 2nd figure. This problem was posed in the 19th International Mathematical Talent Search (IMTS) in 1996, and can solved in several different ways. The following solution is relatively simple, yet elegant.

Construct *HF* and *BD* to intersect at *Q*, the centre of the square. Since *AEGD* is a rectangle, it follows that *P*, the intersection of its diagonals, is the midpoint of *HQ*. Since *HN* and *DQ* are medians of ΔHFD, it follows that $QO = 1/3\ QD$. Therefore the area of $\Delta PQO = ½\ PO \times OQ \times \sin PQO = ½ \times (½\ HQ) \times (1/3\ QD) \times \sin PQO = 1/6\ (½\ HQ \times QD \times \sin PQO) = 1/6$ area of ΔHQD. But the same result, clearly holds going around the other seven triangles into which the octagon can be divided; hence the relationship follows.

This area result can be further generalized to a parallelogram, as well as dividing the sides of the parallelogram into other ratios as shown in De Villiers (1999). An interactive Web ~~*Java*~~*Sketchpad* webpage that illustrates this generalization is available at: http://frink.machighway.com/~dynamicm/imts.html http://dynamicmathematicslearning.com/imts.html

It is hoped that mathematics teachers will consider using the diagram on the 2006 LTM cover to illustrate to their learners, firstly, the danger of just relying on visual

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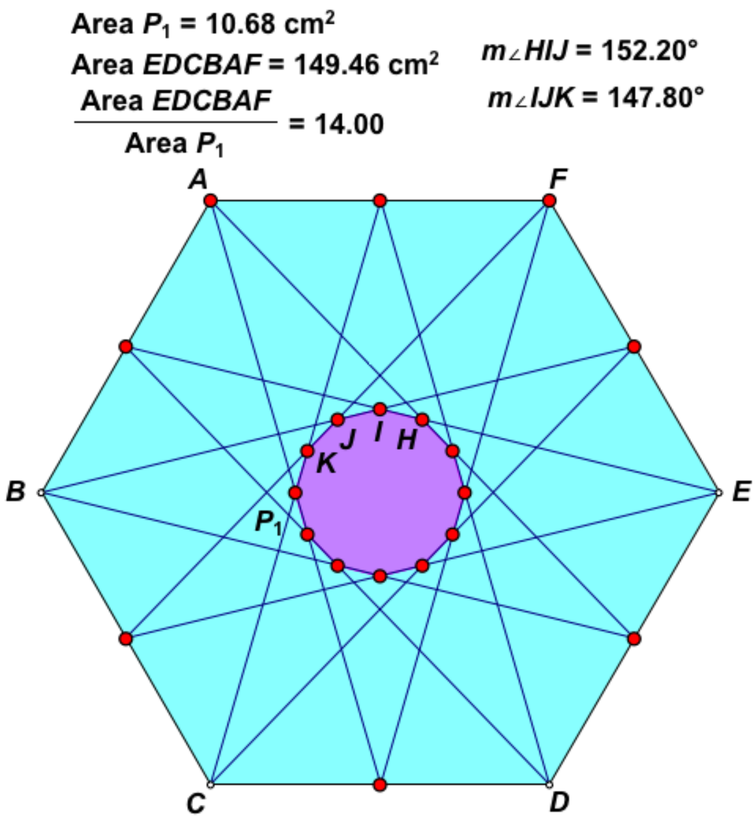
appearance, and secondly, to explore and logically explain (prove) some of the interesting properties of the formed hexagon.



Lastly, it is left to the reader to prove that for the same construction for a regular hexagon as shown above, we obtain a semi-regular side-dodecagon, and that its area is 1/14 that of the original hexagon. An even further challenge is to find a general formula of this area ratio for any regular $2n$-gon and its associated interior semi-regular side-gon.

References

De Villiers, M. (1999). A Generalization of an IMTS Problem. *KZN Mathematics Journal*, 4(1), March, pp. 12-15.

De Villiers, M. (2011). Equi-angled cyclic and equilateral circumscribed polygons. *The Mathematical Gazette*, 95(532), March, pp. 102-106.