

# A Look at High Performance Computing

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# Outline

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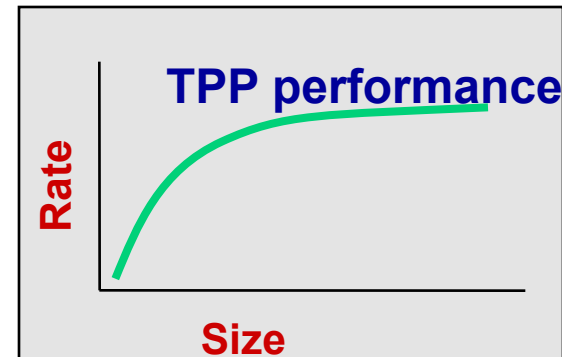
- Overview of High Performance Computing
- With Extreme Computing the “rules” for computing have changed

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H. Meuer, H. Simon, E. Strohmaier, & JD

- Listing of the 500 most powerful Computers in the World
- Yardstick: Rmax from LINPACK MPP

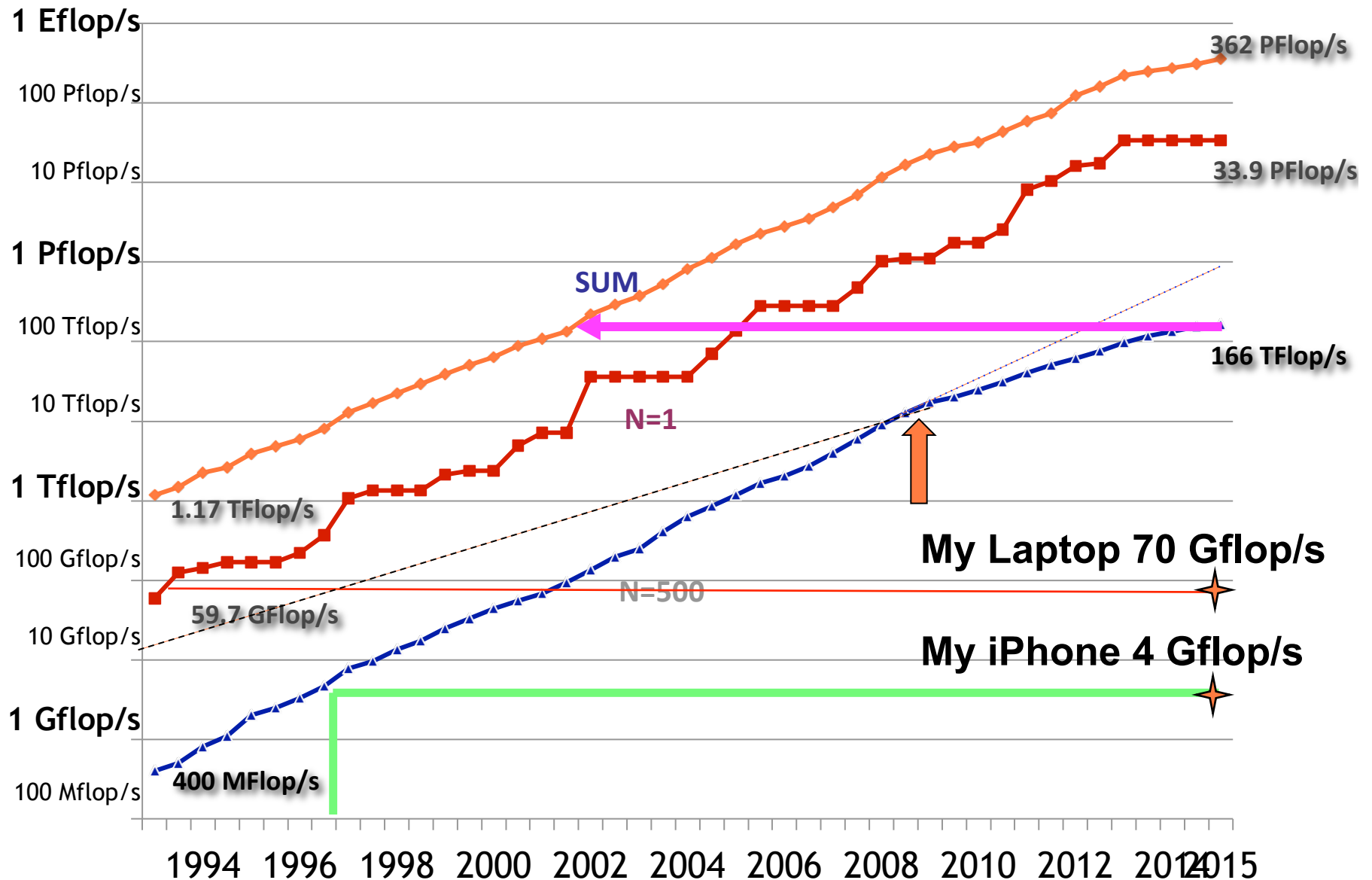
$$Ax=b, \text{ dense problem}$$



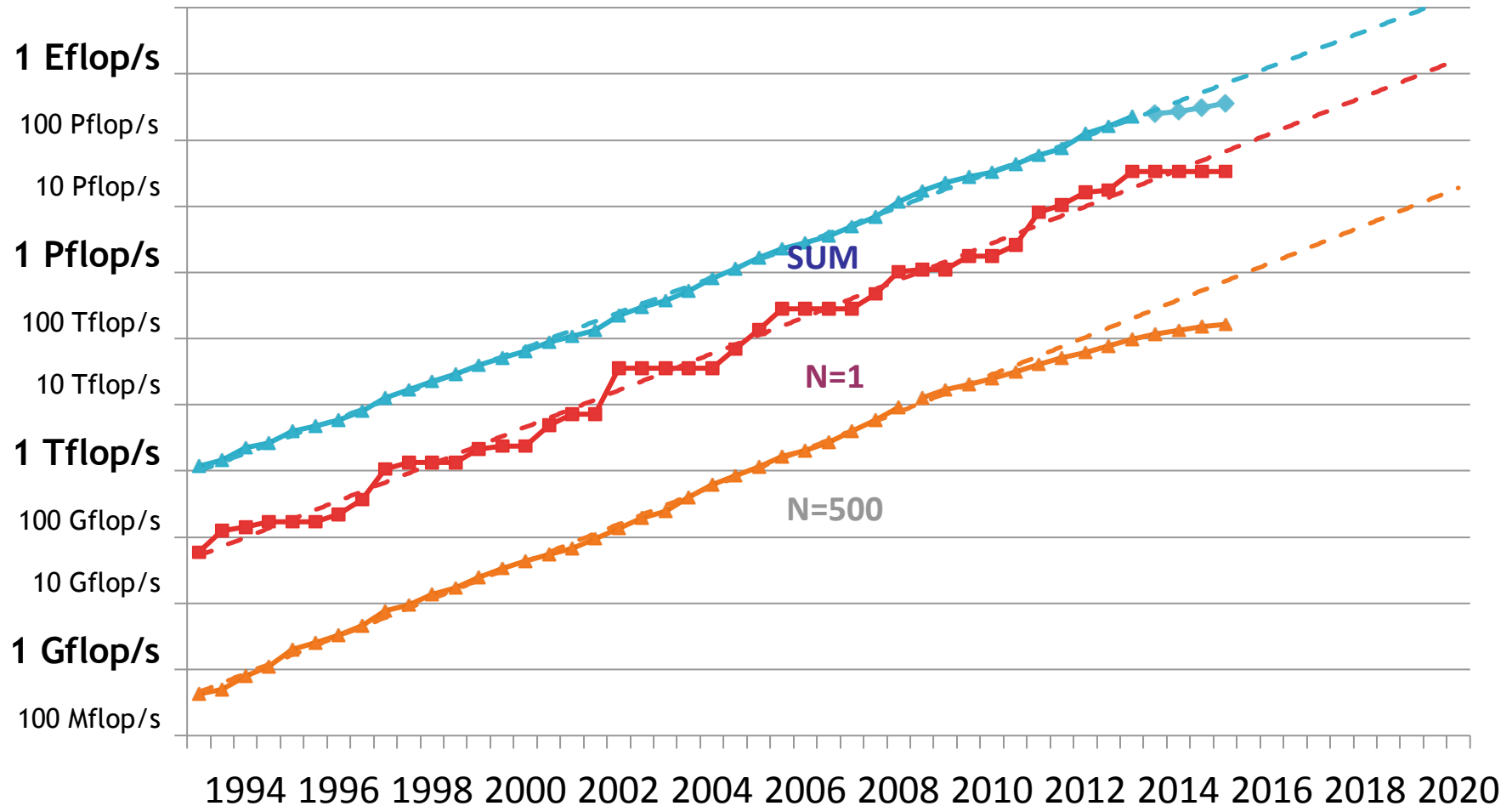
- Updated twice a year  
SC'xy in the States in November  
Meeting in Germany in June
- All data available from [www.top500.org](http://www.top500.org)



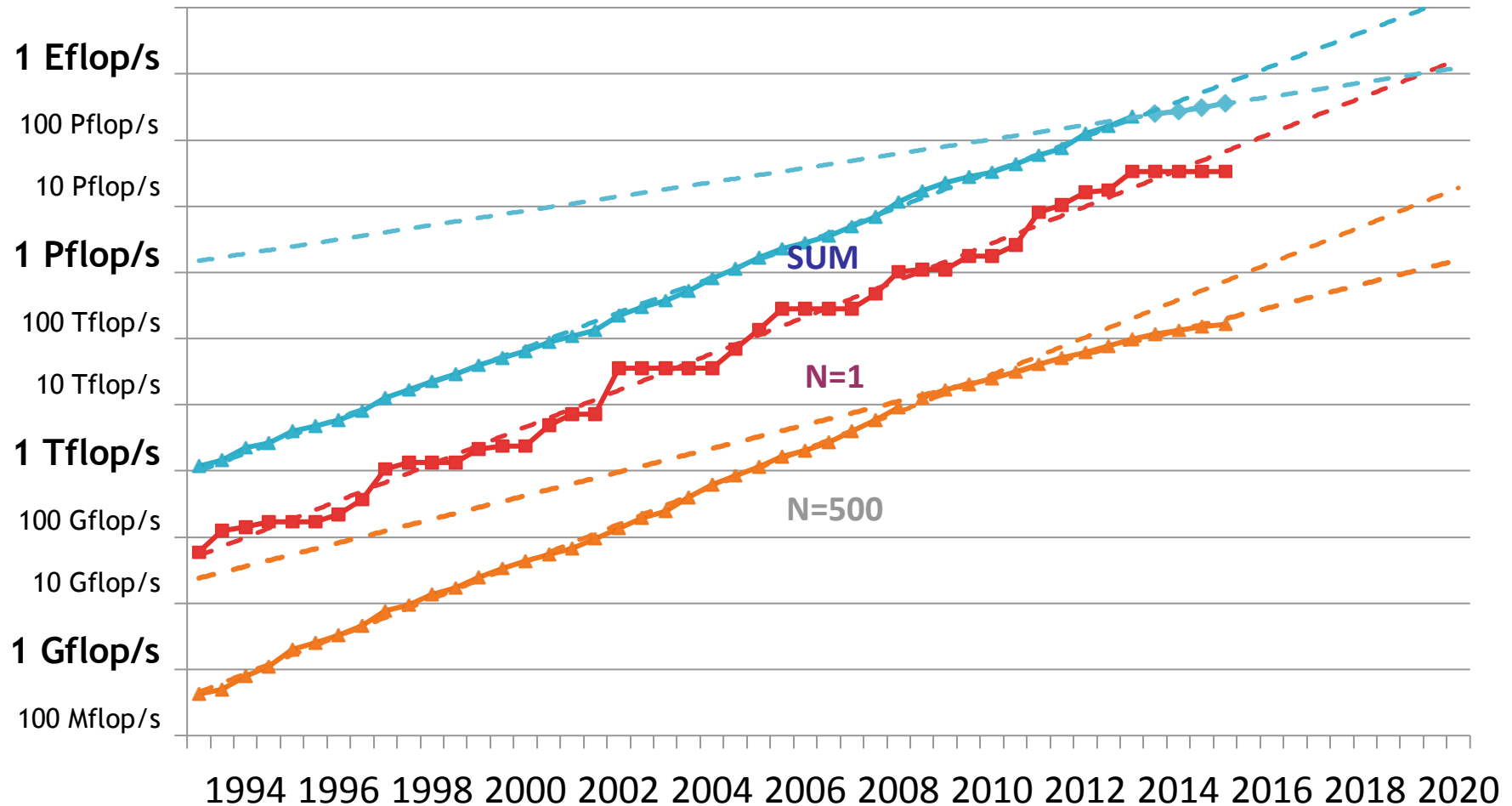
# Performance Development of HPC over the Last 23 Years from the Top500



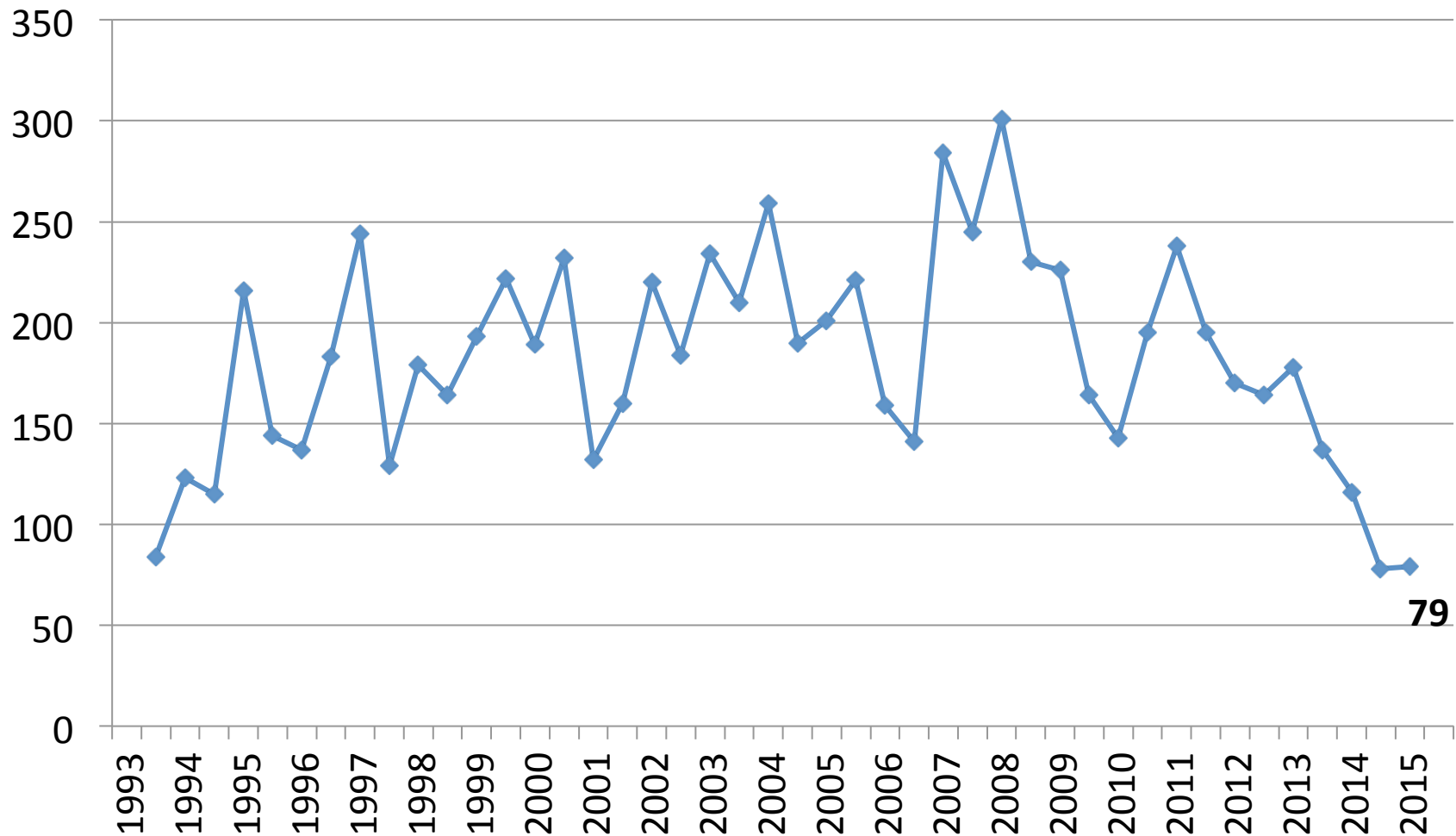
# Projected Performance Development



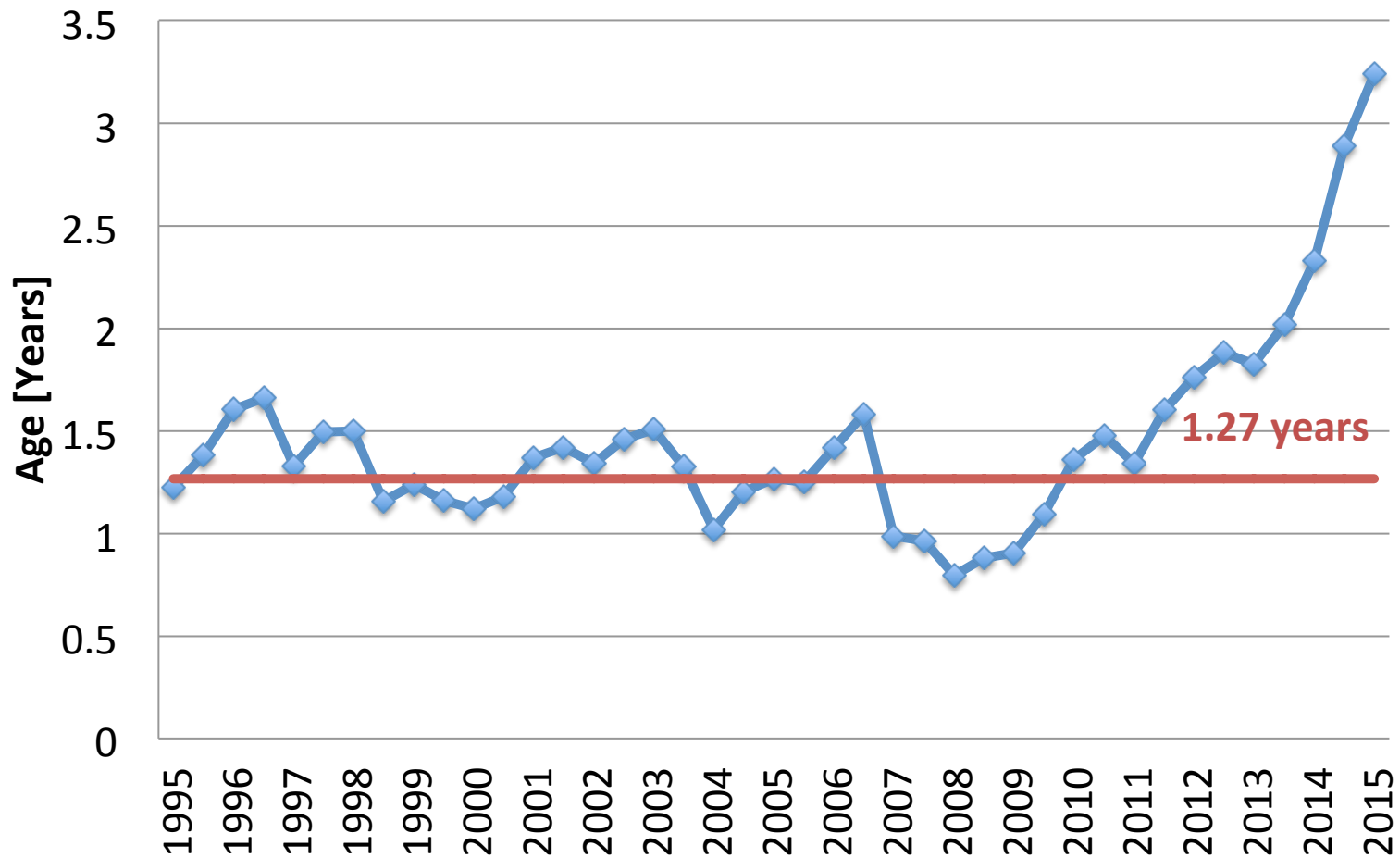
# Projected Performance Development



# Replacement Rate in List



# Average System Age



# State of Supercomputing in 2015

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- Pflops ( $> 10^{15}$  Flop/s) computing fully established with 67 systems.
- Three technology architecture possibilities or “swim lanes” are thriving.
  - Commodity (e.g. Intel)
  - Commodity + accelerator (e.g. GPUs) (88 systems)
  - Special purpose lightweight cores (e.g. IBM BG, ARM, Knights Landing)
- Interest in supercomputing is now worldwide, and growing in many new markets (over 50% of Top500 computers are in industry).
- Exascale ( $10^{18}$  Flop/s) projects exist in many countries and regions.
- Intel processors largest share, 86% followed by AMD, 4%.

# July 2015: The TOP 10 Systems

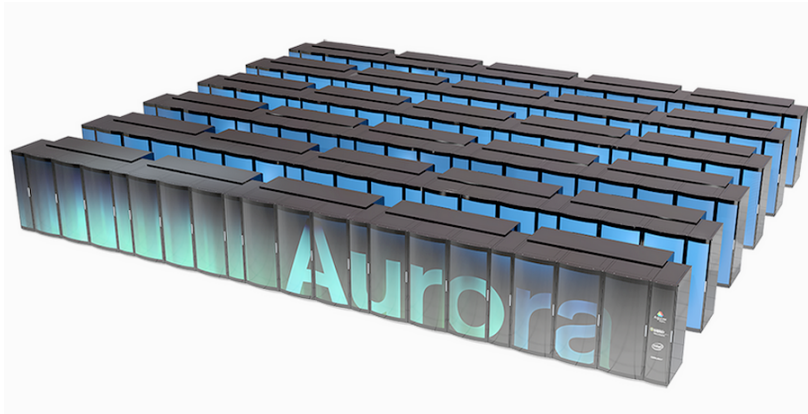
| Rank                    | Site                                        | Computer                                                             | Country                                                                                            | Cores     | Rmax [Pflops] | % of Peak | Power [MW] | MFlops /Watt |
|-------------------------|---------------------------------------------|----------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|-----------|---------------|-----------|------------|--------------|
| 1                       | National Super Computer Center in Guangzhou | Tianhe-2 NUDT, Xeon 12C 2.2GHz + <b>IntelXeon Phi (57c)</b> + Custom |  China          | 3,120,000 | 33.9          | 62        | 17.8       | 1905         |
| 2                       | DOE / OS Oak Ridge Nat Lab                  | Titan, Cray XK7, AMD (16C) + <b>Nvidia Kepler GPU (14c)</b> + Custom |  USA            | 560,640   | 17.6          | 65        | 8.3        | 2120         |
| 3                       | DOE / NNSA L Livermore Nat Lab              | Sequoia, BlueGene/Q (16c) + custom                                   |  USA            | 1,572,864 | 17.2          | 85        | 7.9        | 2063         |
| 4                       | RIKEN Advanced Inst for Comp Sci            | K computer Fujitsu SPARC64 VIIIfx (8c) + Custom                      |  Japan          | 705,024   | 10.5          | 93        | 12.7       | 827          |
| 5                       | DOE / OS Argonne Nat Lab                    | Mira, BlueGene/Q (16c) + Custom                                      |  USA            | 786,432   | 8.16          | 85        | 3.95       | 2066         |
| 6                       | Swiss CSCS                                  | Piz Daint, Cray XC30, Xeon 8C + <b>Nvidia Kepler (14c)</b> + Custom  |  Swiss        | 115,984   | 6.27          | 81        | 2.3        | 2726         |
| 7                       | KAUST                                       | Shaheen II, Cray XC30, Xeon 16C + Custom                             |  Saudi Arabia | 196,608   | 5.54          | 77        | 4.5        | 1146         |
| 8                       | Texas Advanced Computing Center             | Stampede, Dell Intel (8c) + <b>Intel Xeon Phi (61c)</b> + IB         |  USA          | 204,900   | 5.17          | 61        | 4.5        | 1489         |
| 9                       | Forschungszentrum Juelich (FZJ)             | JuQUEEN, BlueGene/Q, Power BQC 16C 1.6GHz+Custom                     |  Germany      | 458,752   | 5.01          | 85        | 2.30       | 2178         |
| 10                      | DOE / NNSA L Livermore Nat Lab              | Vulcan, BlueGene/Q, Power BQC 16C 1.6GHz+Custom                      |  USA          | 393,216   | 4.29          | 85        | 1.97       | 2177         |
| 500 (422) Software Comp |                                             | HP Cluster                                                           | USA                                                                                                | 18,896    | .309          | 48        |            |              |

# Eight Russian Top500 Systems

| Rank | Name                       | Site                                         | Manufacturer                        | Cores  | Rmax<br>(Pflop/s) | Rpeak<br>(Pflop/s) | Acc    |
|------|----------------------------|----------------------------------------------|-------------------------------------|--------|-------------------|--------------------|--------|
| 30   | Lomonosov-2                | MSU                                          | T-Platforms                         | 37,120 | 1.84              | 2.57               | Nvidia |
| 77   | Lomonosov                  | MSU                                          | T-Platforms                         | 78,660 | .901              | 1.70               | Nvidia |
| 106  | Polytechnic RSC<br>Tornado | St. Petersburg<br>Polytechnic U              | RSC Group                           | 19,936 | .658              | .829               | none   |
| 176  | MVS-10P                    | Joint<br>Supercomputer<br>Center             | RSC Group                           | 28,704 | .375              | .523               | Phi    |
| 242  | Lobachevsky                | Lobachevsky<br>State U of<br>Nizhni Novgorod | Niagara<br>Computers,<br>Supermicro | 5310   | .289              | .348               | Nvidia |
| 245  | RSC Tornado<br>SUSU        | South Ural<br>State University               | RSC Group                           | 28,032 | .288              | .473               | Phi    |
| 416  |                            | IT Services<br>Provider                      | Hewlett-<br>Packard                 | 13,180 | .189              | .295               | none   |
| 469  | RSC PetaStream             | St. Petersburg<br>Polytechnic U              | RSC Group                           | 22,528 | .170              | .572               | Phi    |

# Recent Developments

- .. US DOE planning to deploy O(100) Pflop/s systems for 2017-2018 - \$525M hardware
- .. Oak Ridge Lab and Lawrence Livermore Lab to receive IBM and Nvidia based systems
- .. Argonne Lab to receive Intel based system
  - **After this Exaflops**



# Peak Performance - Per Core

$$\text{FLOPS} = \text{cores} \times \text{clock} \times \frac{\text{FLOPs}}{\text{cycle}}$$

## Floating point operations per cycle per core

- + Most of the recent computers have FMA (Fused multiple add): (i.e.  $x \leftarrow x + y * z$  in one cycle)
- + Intel Xeon earlier models and AMD Opteron have SSE2
  - + 2 flops/cycle DP & 4 flops/cycle SP
- + Intel Xeon Nehalem ('09) & Westmere ('10) have SSE4
  - + 4 flops/cycle DP & 8 flops/cycle SP
- + Intel Xeon Sandy Bridge('11) & Ivy Bridge ('12) have AVX
  - + 8 flops/cycle DP & 16 flops/cycle SP
- + Intel Xeon Haswell ('13) & (Broadwell ('14)) AVX2
  - + 16 flops/cycle DP & 32 flops/cycle SP
- + Xeon Phi (per core) is at 16 flops/cycle DP & 32 flops/cycle SP
- + Intel Xeon Skylake ('15)
  - + 32 flops/cycle DL & 64 flops/cycle SP



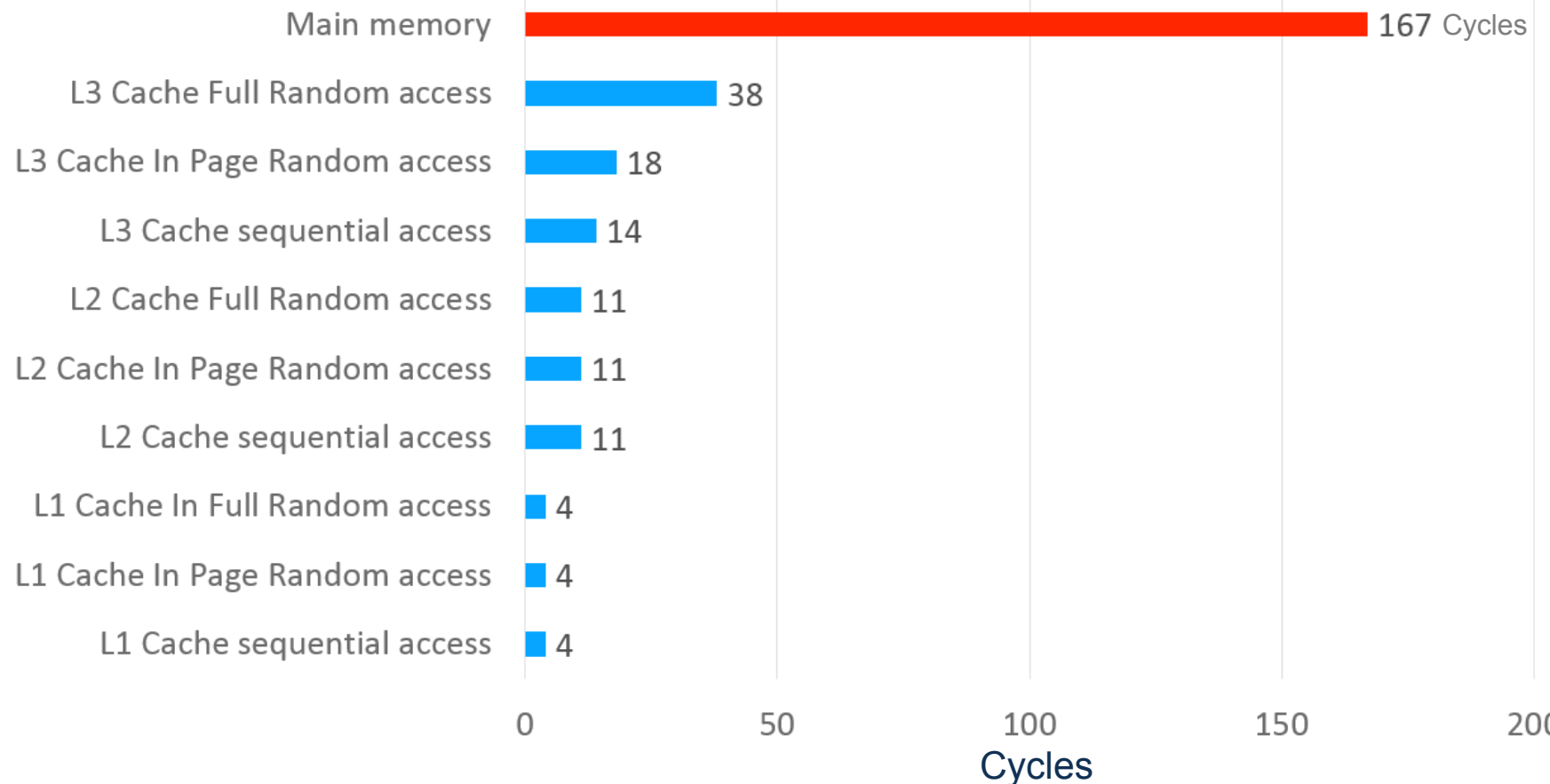
We  
are  
here



|                                 |                              |                              |                               |                             |                               |
|---------------------------------|------------------------------|------------------------------|-------------------------------|-----------------------------|-------------------------------|
| 87 GFLOPS<br>(DP-FP, peak)      | 185 GFLOPS<br>(DP-FP, peak)  | ~225 GFLOPS<br>(DP-FP, peak) | ~500 GFLOPS<br>(DP-FP, peak)  | 1tb GFLOPS<br>(DP-FP, peak) | 1tb GFLOPS<br>(DP-FP, peak)   |
| Westmere                        | Sandy Bridge                 | Ivy Bridge                   | Haswell                       | Broadwell                   | Skylake                       |
| 32nm<br>SSE4.2<br>DDR3<br>PCIe2 | 32nm<br>AVX<br>DDR3<br>PCIe3 | 22nm                         | 22nm<br>AVX2<br>DDR4<br>PCIe3 | 14nm                        | 14nm<br>AVX2<br>DDR4<br>PCIe4 |

# CPU Access Latencies in Clock Cycles

In 167 cycles can do 2672 DP Flops



# Classical Analysis of Algorithms May Not be Valid

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- Processors over provisioned for floating point arithmetic
- Data movement extremely expensive
- Operation count is not a good indicator of the time to solve a problem.
- Algorithms that do more ops may actually take less time.

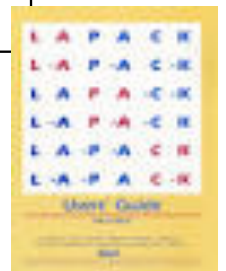
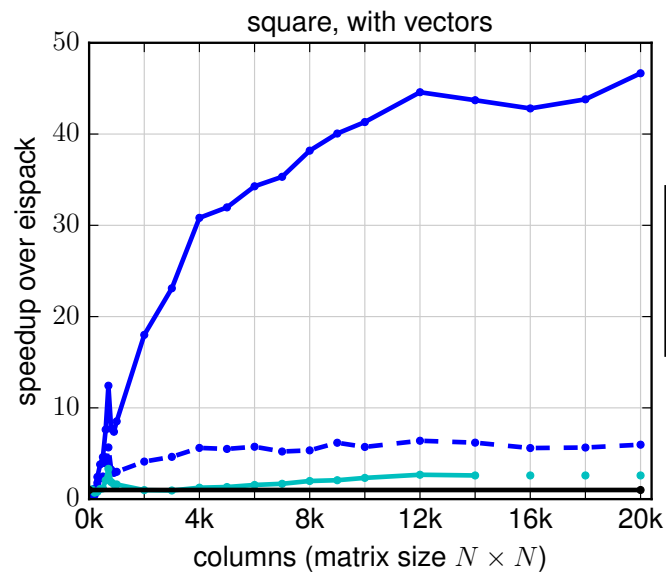
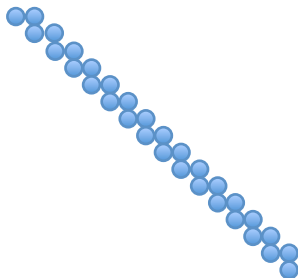
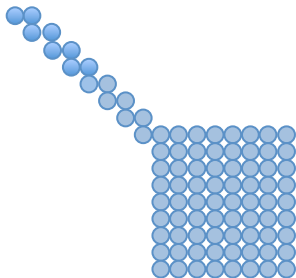
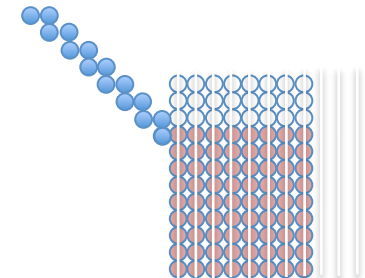
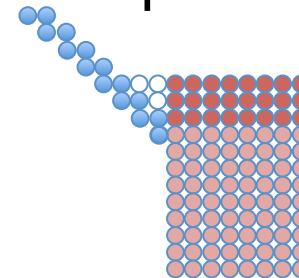
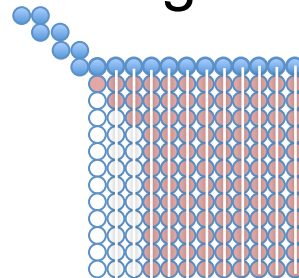
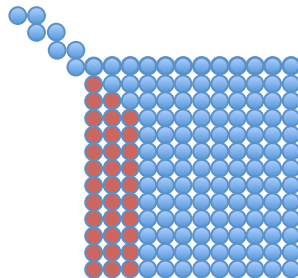
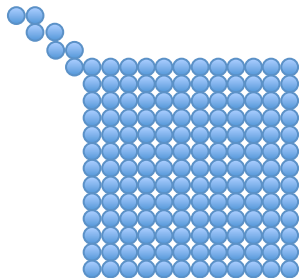
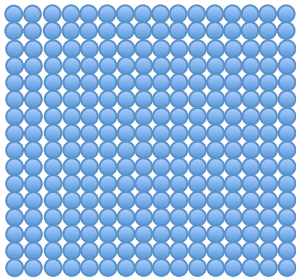


# Singular Value Decomposition

## LAPACK Version 1991

Level 1, 2, & 3 BLAS

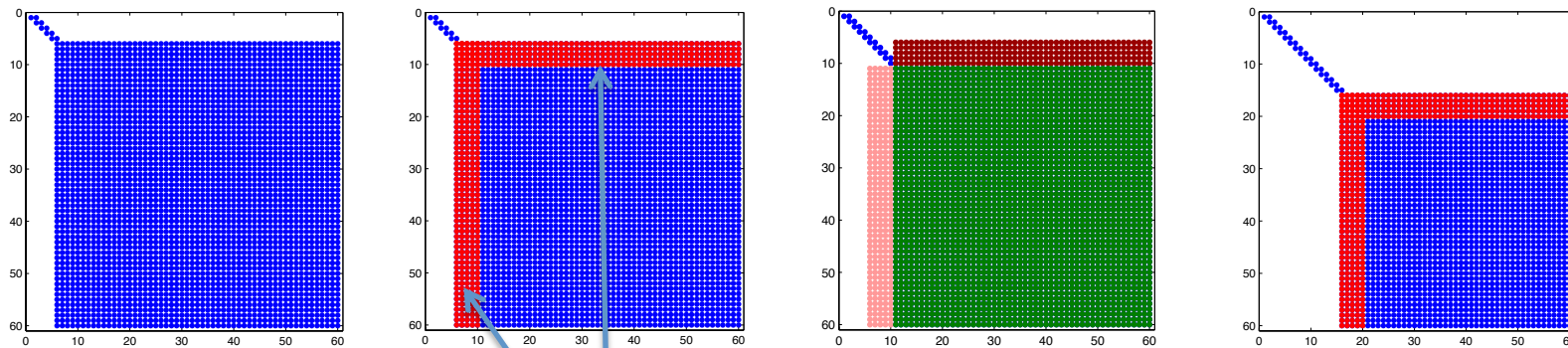
First Stage  $\frac{8}{3} n^3$  Ops



# Bottleneck in the Bidiagonalization

## The Standard Bidiagonal Reduction: xGEBRD

### Two Steps: Factor Panel & Update Tailing Matrix

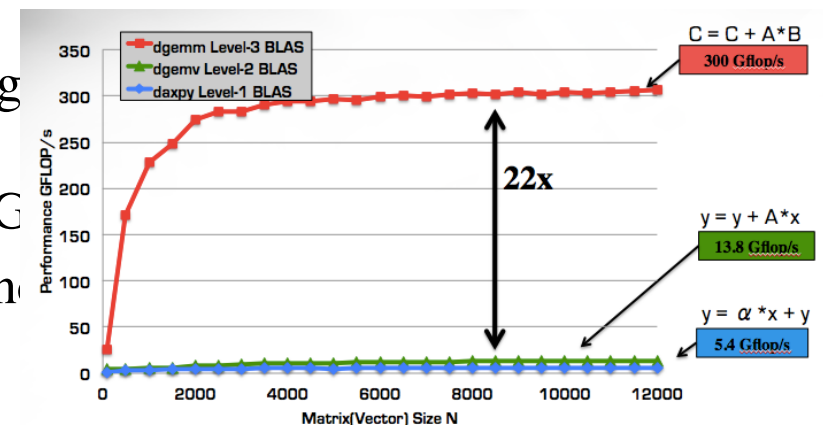


factor panel k  
Requires 2 GEMVs

then update → factor panel k+1  
 $Q * A * P^H$

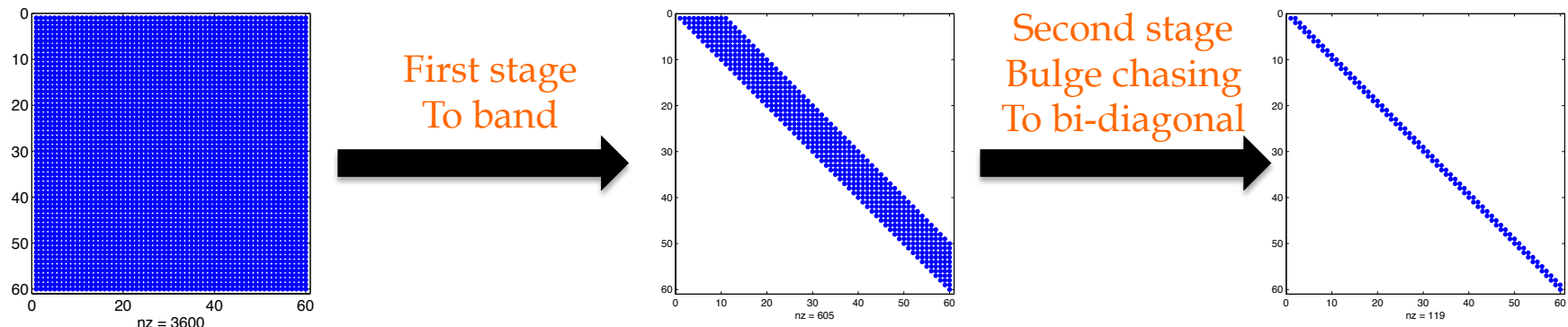
## ★ Characteristics

- Total cost  $8n^3/3$ , (reduction to bi-diag)
- Too many Level 2 BLAS operations
- $4/3 n^3$  from GEMV and  $4/3 n^3$  from G
- Performance limited to 2\* performan
- → **Memory bound algorithm.**



16 cores Intel Sandy Bridge, 2.6 GHz, 20 MB shared L3 cache.  
The theoretical peak per core double precision is 20.4 Gflop/s per core.  
Compiled with icc and using MKL 2015.3.187

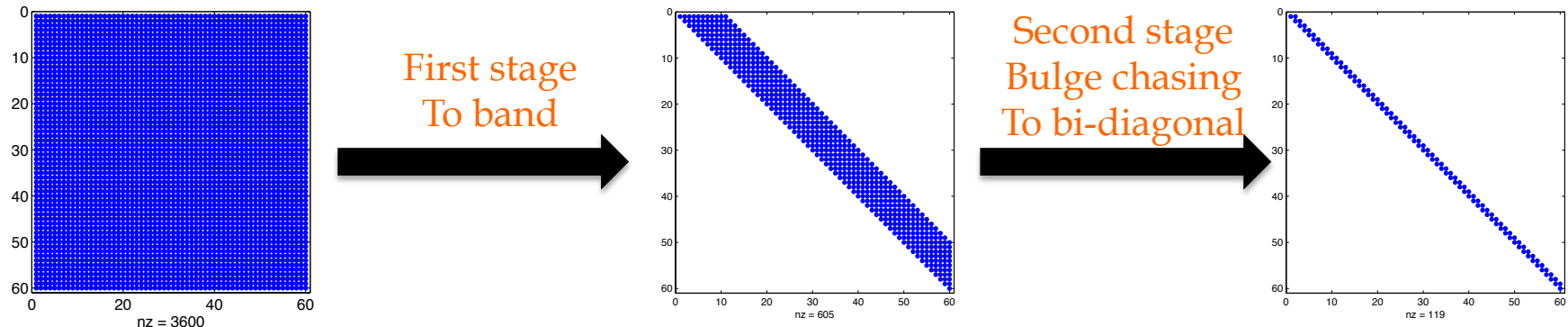
# Recent Work on 2-Stage Algorithm



## ★ Characteristics

- **Stage 1:**
  - Fully Level 3 BLAS
  - Dataflow Asynchronous execution
- **Stage 2:**
  - Level “BLAS-1.5”
  - Asynchronous execution
  - Cache friendly kernel (reduced communication)

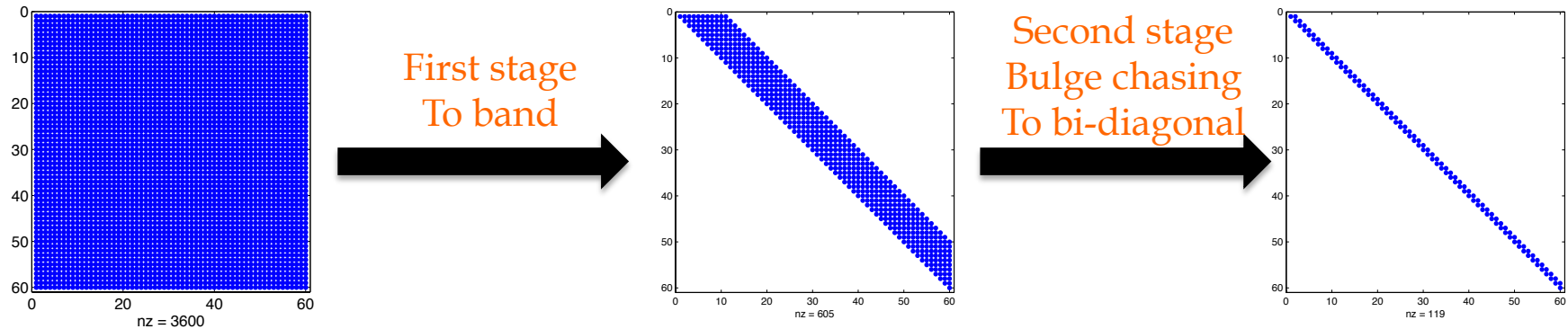
# Recent work on developing new 2-stage algorithm



$$\begin{aligned}
 \text{flops} &\approx \sum_{s=1}^{\frac{n-n_b}{n_b}} 2n_b^3 + (nt-s)3n_b^3 + (nt-s)\frac{10}{3}n_b^3 + (nt-s) \times (nt-s)5n_b^3 \\
 &+ \sum_{s=1}^{\frac{n-n_b}{n_b}} 2n_b^3 + (nt-s-1)3n_b^3 + (nt-s-1)\frac{10}{3}n_b^3 + (nt-s) \times (nt-s-1)5n_b^3 \\
 &\approx \frac{10}{3}n^3 + \frac{10n_b}{3}n^2 + \frac{2n_b}{3}n^3 \\
 &\approx \frac{10}{3}n^3(\text{gemm})_{\text{first stage}} \qquad \text{flops} = 6 \times n_b \times n^2(\text{gemv})_{\text{second stage}}
 \end{aligned}$$

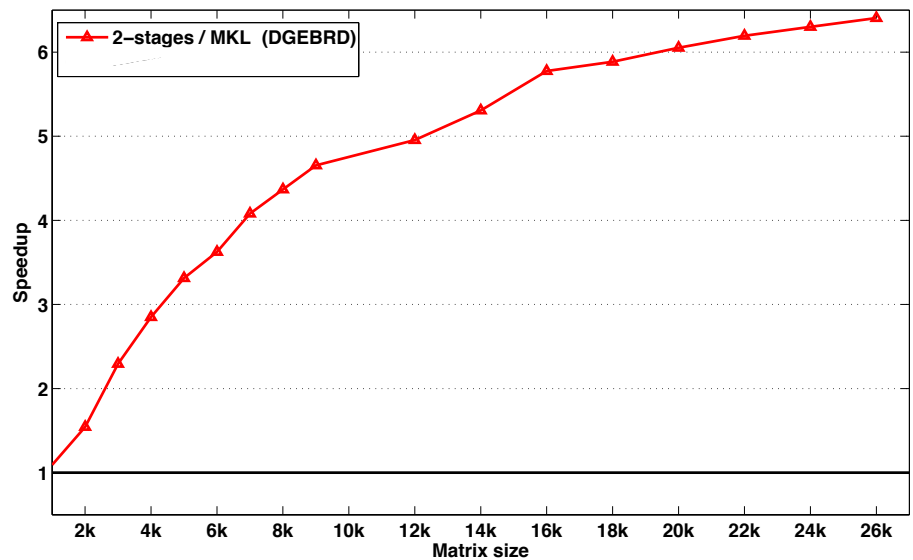
More Flops, original did  $\frac{8}{3} n^3$

# Recent work on developing new 2-stage algorithm



$$\begin{aligned}
 \text{speedup} &= \frac{\text{time of one-stage}}{\text{time of two-stage}} \\
 &= \frac{4n^3/3P_{\text{gemv}} + 4n^3/3P_{\text{gemm}}}{10n^3/3P_{\text{gemm}} + 6n_b n^2/P_{\text{gemv}}} \\
 &\Rightarrow \frac{84}{70} \leq \text{Speedup} \leq \frac{84}{15} \\
 &\Rightarrow 1.8 \leq \text{Speedup} \leq 7
 \end{aligned}$$

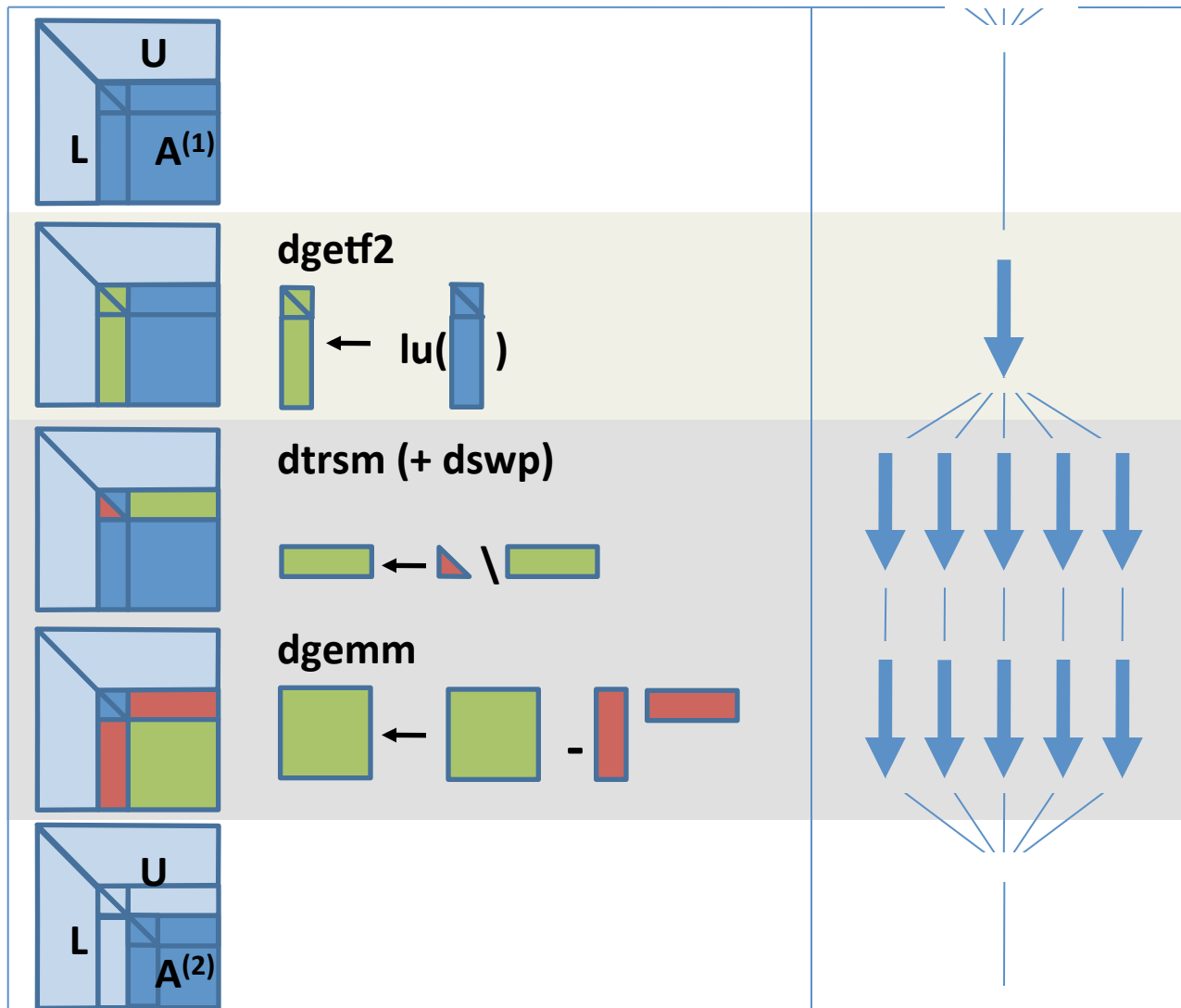
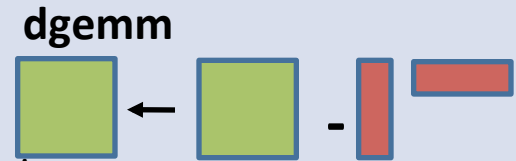
if  $P_{\text{gemm}}$  is about 22x  $P_{\text{gemv}}$  and  $120 \leq n_b \leq 240$ .



## Parallelization of LU and QR.

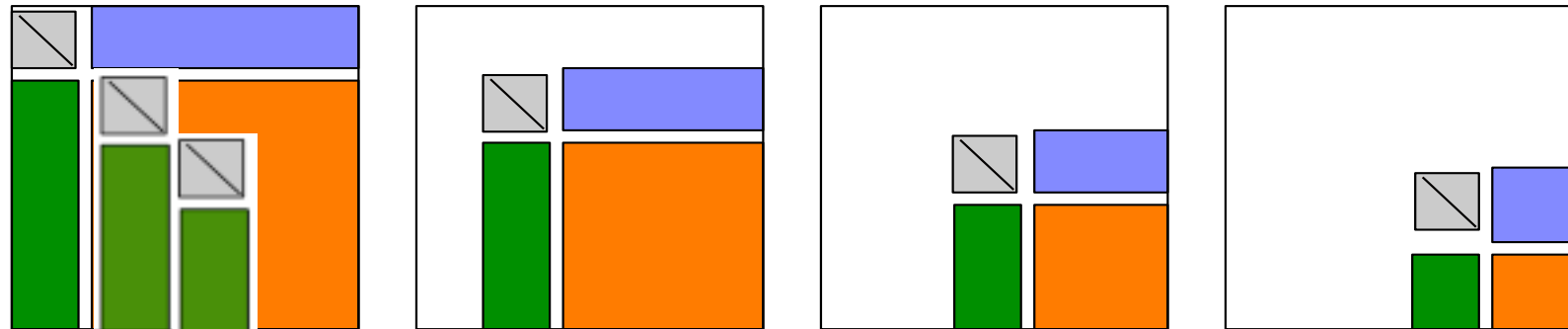
### Parallelize the update:

- Easy and done in any reasonable software.
- This is the  $2/3n^3$  term in the FLOPs count.
- Can be done efficiently with LAPACK+multithreaded BLAS

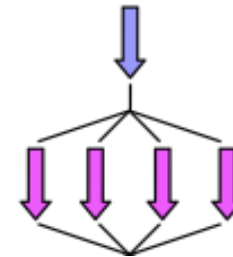
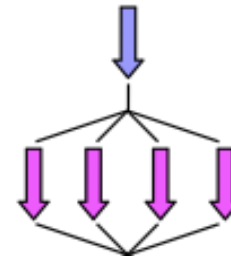
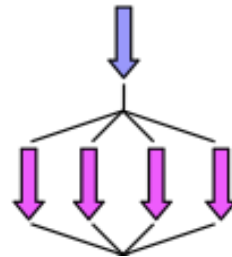
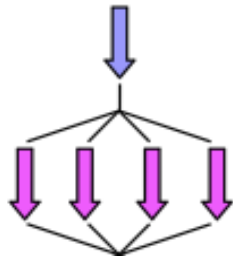


Fork - Join parallelism  
Bulk Sync Processing

# Synchronization (in LAPACK LU)



Step 1 → Step 2 → Step 3 → Step 4 . . .



DGETF2  
(Factor a panel)



DLSWP  
(Backward swap)



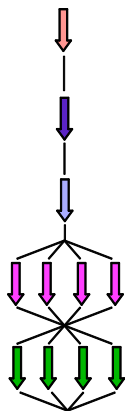
DLSWP  
(Forward swap)



DTRSM  
(Triangular solve)



DGEMM  
(Matrix multiply)



LAPACK

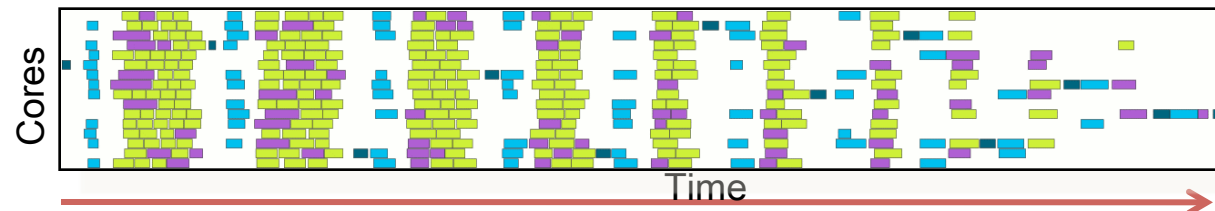
LAPACK

LAPACK

BLAS

BLAS

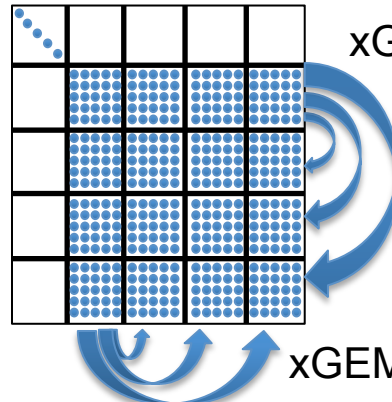
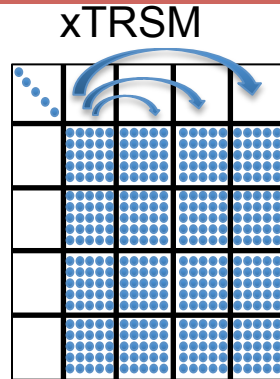
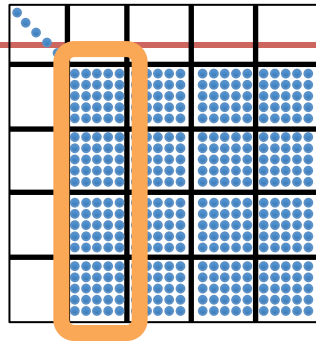
- fork join
- bulk synchronous processing



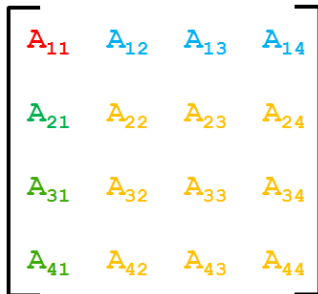
# PLASMA LU Factorization

## Dataflow Driven

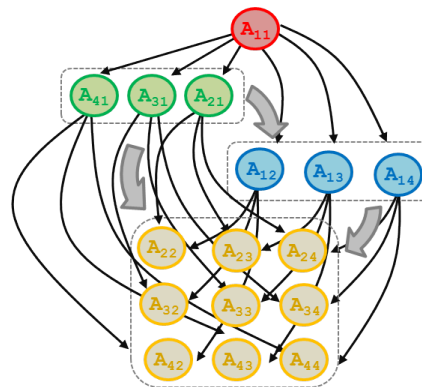
Numerical program generates tasks and run time system executes tasks respecting data dependences.



### Sparse / Dense Matrix System



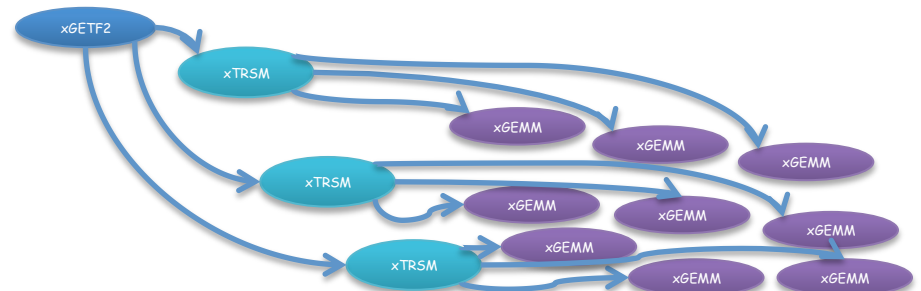
### DAG-based factorization



### Batched LA

- ➔ • LU, QR, or Cholesky on small diagonal matrices
- ➔ • TRSMs, QRs, or LUs
- ➔ • TRSMs, TRMMs
- ➔ • Updates (Schur complement) GEMMs, SYRKs, TRMMs

And many other BLAS/LAPACK, e.g., for application specific solvers, preconditioners, and matrices



# Dataflow Based Design

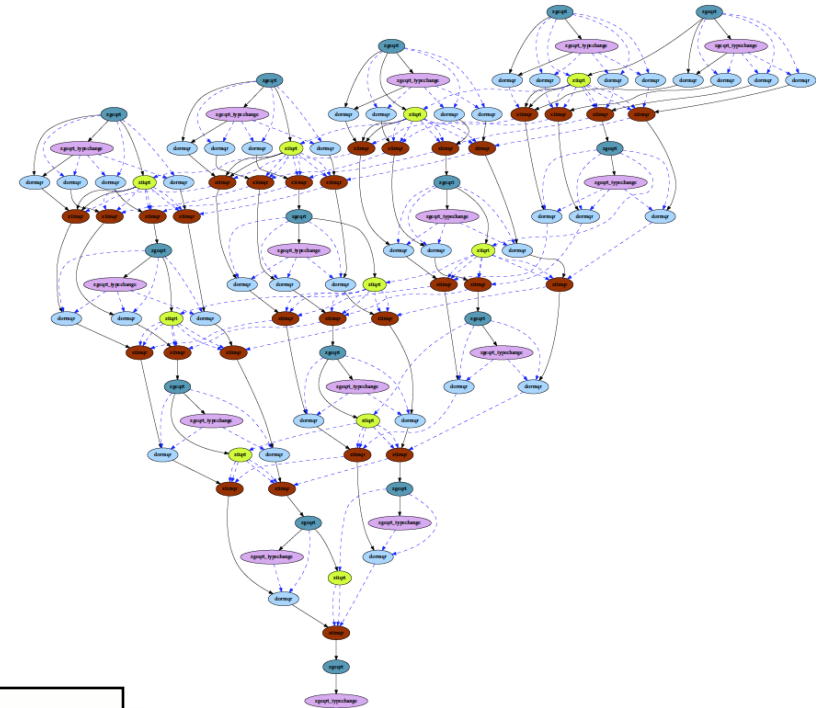
## Objectives

- High utilization of each core
- Scaling to large number of cores
- Synchronization reducing algorithms

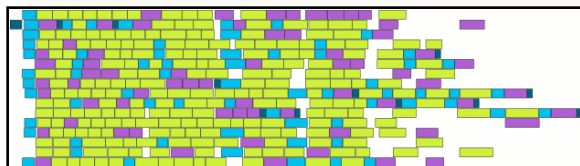
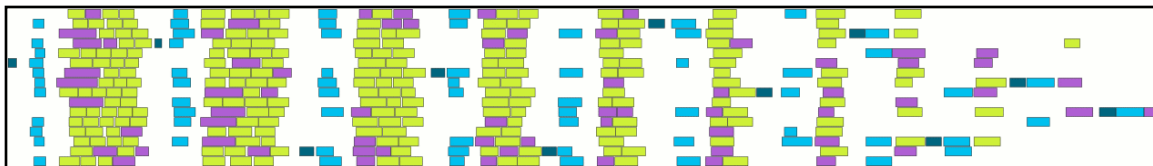
## Methodology

- Dynamic DAG scheduling
- Explicit parallelism
- Implicit communication
- Fine granularity / block data layout

## Arbitrary DAG with dynamic scheduling



Fork-join parallelism  
Notice the synchronization penalty in the presence of heterogeneity.



DAG scheduled parallelism

# Algorithmic Bombardment

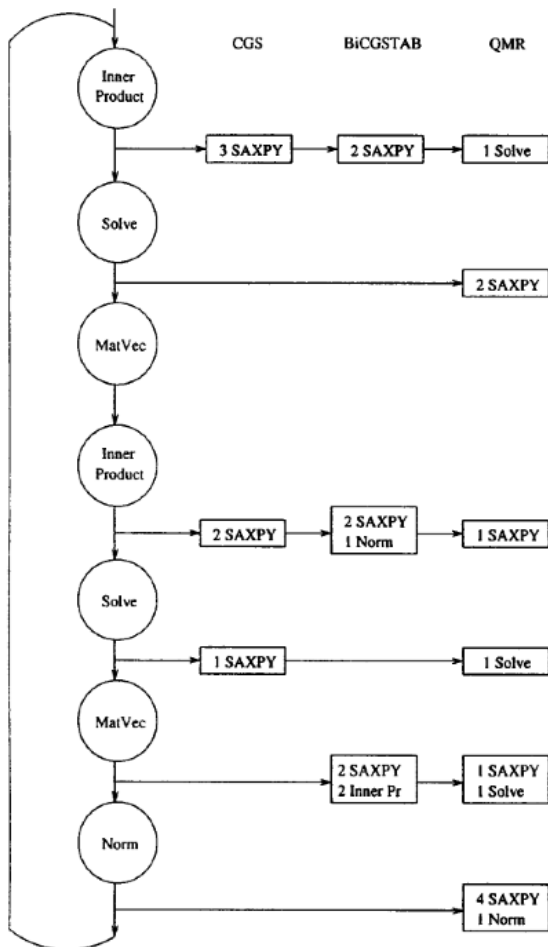
- Interesting in solving a large sparse non-symmetric system
- system on a parallel computer.
- Many algorithms to choose
  - CG Square
  - Bi CG Stabilized
  - QMR
  - GMRES
  - etc.
- Best choice not clear
- Some may not converge
- Develop a “poly-algorithm” combining multiple algorithms in one, leveraging data movement.

# Poly-algorithm

## Iterating in Lock Step

Amount of work per iteration

| Method    | $\alpha = x^T y$ | $y = \alpha x + y$ | $y = Ax$ |
|-----------|------------------|--------------------|----------|
| CGS       | 2                | 6                  | 2        |
| Bi-CGSTAB | 4                | 6                  | 2        |
| QMR       | 2                | 12                 | 2        |



Number of Communications per iteration

| Method      | $\alpha = x^T y$ | $y = \alpha x + y$ | $y = Ax$ | $x = M^{-1}y$ | Storage   |
|-------------|------------------|--------------------|----------|---------------|-----------|
| CGS         | 2                | 0                  | 2        | 2             | $A + 6n$  |
| Bi-CGSTAB   | 3                | 0                  | 2        | 2             | $A + 6n$  |
| QMR         | 2                | 0                  | 2        | 2             | $A + 16n$ |
| Bombardment | 3                | 0                  | 2        | 3             | $A + 26n$ |

# Avoiding Synchronization

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## .. “Responsibly Reckless” Algorithms

- Try fast algorithm (unstable algorithm) that might fail (but rarely)
- Check for instability
- If needed, recompute with stable algorithm

# Avoid Pivoting; Minimize Synchronization

## To solve $Ax = b$ :

- Compute  $A_r = U^T A V$ , with  $U$  and  $V$  random matrices
- Factorize  $A_r$  without pivoting (GENP)
- Solve  $A_r y = U^T b$  and then Solve  $x = Vy$

## $U$ and $V$ are Recursive Butterfly Matrices

- Randomization is cheap ( $O(n)$  operations)
- GENP is fast (can take advantage of the GPU)
- Accuracy is in practice similar to GEPP (with iterative refinement), but...

Think of this as a preconditioner step.

Goal: Transform  $A$  into a matrix that would be sufficiently “random” so that, with a probability close to 1, pivoting is not needed.

A **butterfly matrix** is defined as any  $n$ -by- $n$  matrix of the form:

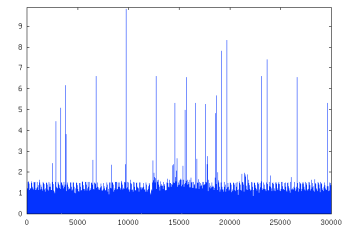
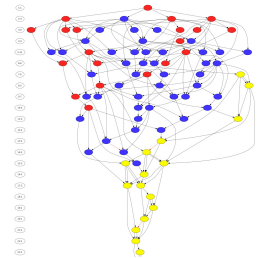
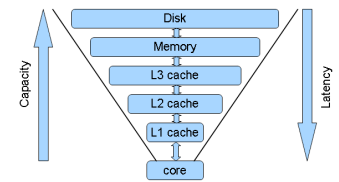
$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} R & S \\ R & -S \end{pmatrix}$$

where  $R$  and  $S$  are random diagonal matrices.

$$B = \begin{pmatrix} \text{red} & \text{green} \\ \text{red} & \text{green} \end{pmatrix}$$

# Software and Algorithm Must Keep Pace with the Changes in Hardware

- Classical analysis of algorithms is not valid,
  - # of floating point ops  $\neq$  computation time.
- Algorithms and software must take advantage by reducing data movement.
  - Need latency tolerance in our algorithms
- Communication and synchronization reducing algorithms and software are critical.
  - As parallelism grows
- Hardware presents a dynamically changing environment
  - Turbo Boost and OS jitter
- Many existing algorithms can't fully exploit the features of modern architecture



# Summary

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- **Major Challenges are ahead for extreme computing**
  - **Parallelism  $O(10^9)$** 
    - Programming issues
  - **Hybrid**
    - Peak and HPL may be very misleading
    - No where near close to peak for most apps
  - **Fault Tolerance**
    - Today Sequoia BG/Q node failure rate is 1.25 failures/day
  - **Power**
    - 50 Gflops/w (today at 2 Gflops/w)
- **We will need completely new approaches and technologies to reach the Exascale level**