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“Relative Allocation, Protection, and Excess Investment”

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Abstract

This paper develops a structural theory of investment under dual incentive structures. Investment affects economic outcomes through two distinct channels. First, it increases aggregate protection against external threats and enlarges total feasible surplus through an increasing defense technology. Second, it improves relative standing within an allocation rule that redistributes the resulting surplus across agents. When investment operates through both channels, private and social incentives need not coincide. In a symmetric environment with monotone relative allocation, I show that equilibrium investment exceeds the social optimum whenever comparative screening intensity exceeds the critical ratio of marginal aggregate protection to average protected surplus. Overinvestment therefore does not arise because protection lacks social value; it arises because relative evaluation adds a positional component to private returns without increasing aggregate surplus. The paper further shows that transparency acts as an amplification parameter, while public standards mitigate escalation by reducing the granularity with which investment is translated into relative ranking. The analysis unifies ideas from contests, status competition, public information, and incentive design, and identifies a general mechanism of protection-compatible escalation.

JEL Classification: D60, D82, D83

Keywords: Relative Allocation, Excess Investment, Positional Incentives, Transparency, Screening, Regulatory Design

1. Introduction

Investment in modern economic environments often operates through two structurally distinct channels. On the one hand, it raises aggregate resilience against external threats, thereby expanding the set of feasible transactions or total surplus. On the other hand, it improves an agent's relative position within allocation mechanisms that distribute that surplus. Once both channels are present, private and social incentives need not coincide. This paper develops a general theory of such dual-channel investment and shows that socially productive protection can nevertheless generate excessive private escalation.

The basic idea is simple. Aggregate investment raises total surplus through a protection technology $T(\bar{x})$. At the same time, allocation across agents depends on a relative rule $\sigma_i(x; \theta)$, where θ measures the intensity of comparative screening. The social planner internalizes only the aggregate protection effect. Individual firms internalize both the aggregate effect and the marginal gain from improving relative position. Because relative allocation redistributes surplus without increasing its aggregate level, it adds a positional component to private returns. The result is a structural wedge between private and social marginal incentives.

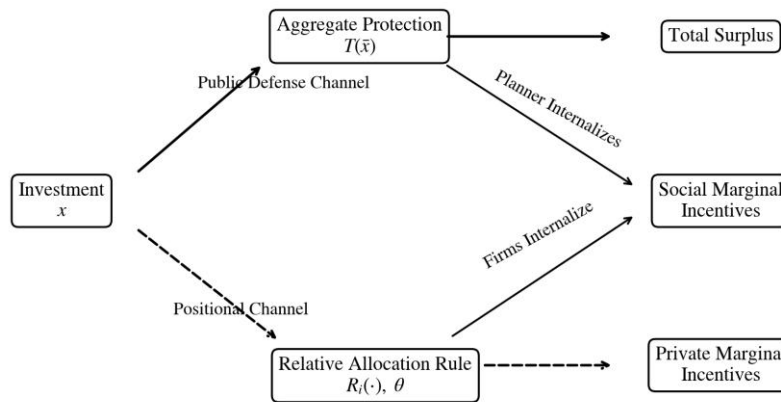
The present framework is deliberately written at a level of abstraction that does not depend on a specific contest success function. Classical contest models (Tullock, 1980; Skaperdas, 1996) and rank-order tournaments (Lazear and Rosen, 1981; Dixit, 1987) appear here only as special cases of a broader relative allocation rule. The argument is therefore not that firms participate in a literal contest. Rather, whenever allocation depends on relative evaluation, screening, or ranking, privately chosen investment may be distorted even if the investment itself is socially valuable. This general formulation delivers a particularly transparent characterization in the symmetric benchmark. There, the private-social wedge can be summarized by a simple threshold condition comparing comparative screening intensity with the marginal effectiveness of aggregate protection.

This perspective links several strands of literature. The positional component relates to status competition and rent-seeking models in which relative advantage shapes incentives (Frank, 1985; Hirshleifer, 1991; Murphy, Shleifer, and Vishny, 1993). Yet unlike a pure zero-sum contest, the present model contains a socially productive defense channel. The informational dimension relates to the social value of public information and information design (Morris and Shin, 2002; Angeletos and Pavan, 2007; Bergemann and Morris, 2019). In the present setting, transparency scales the sensitivity of relative allocation to marginal differences without changing the aggregate protection channel. The regulatory dimension connects to incentive design and multitask problems (Holmström and Milgrom, 1991; Laffont and Tirole, 1993), because public standards modify the mapping from investment to relative evaluation rather than suppressing competition itself.

Cybersecurity investment offers a prominent illustration. Firms invest in genuine protection against cyberattacks or compliance failures, so aggregate investment can raise systemic resilience. At the same time, access to digital platforms, financial networks, procurement chains, and certification tiers increasingly depends on relative evaluation through audits, ratings, rankings, and benchmarks. The same logic extends well beyond cybersecurity to certification systems, disclosure regimes, ESG scorecards, and compliance standards. In each case, investment is both defensive and positional.

The core result is therefore a threshold theorem for the symmetric benchmark. In symmetric equilibrium, equilibrium investment exceeds the social optimum whenever comparative screening intensity exceeds the critical ratio $T'(x^*)/T(x^*)$. This condition identifies the point at which socially productive protection becomes privately over-supplied. The mechanism is not the absence of social value. On the contrary, aggregate defense remains socially productive throughout. The distortion emerges because relative evaluation adds a positional component to private returns without increasing aggregate surplus. The paper also yields two further implications. First, transparency acts as an amplification parameter. As the economy becomes more transparent, the positional component of investment becomes more salient, and the equilibrium may cross from underinvestment to overinvestment. Second, public standards can mitigate excessive escalation by reducing the granularity with which investment differences translate into relative ranking. In that sense, efficient regulation separates the aggregate protection channel from the relative allocation channel.

Figure 1 summarizes the basic structure. Investment affects total surplus through the aggregate protection channel and affects private incentives through the relative allocation channel. The planner internalizes only the former, whereas firms internalize both.



Solid Arrows: Aggregate Protection Channel. Dashed Arrows: Relative Allocation Channel.

Figure 1. Dual-Channel Structure of Investment

The rest of the paper proceeds as follows. Section 2 sets up the model. Section 3 derives the equilibrium condition and presents the main overinvestment theorem. Section 4 studies transparency as an amplification parameter. Section 5 analyzes public standards as a choice of granularity in evaluation. Section 6 decomposes welfare and clarifies the source of distortion. Section 7 concludes. Appendix A shows robustness to labor adjustment frictions, and Appendix B interprets the static threshold condition under regime shifts.

2. Model

2.1 Environment

Consider a static economy with $N \geq 2$ symmetric firms indexed by $i = 1, \dots, N$. Each firm chooses a nonnegative investment level $x_i \in [0, \infty)$. The investment may be interpreted as security investment, compliance expenditure, certification effort, or, more generally, any costly activity that both raises aggregate protection and affects relative evaluation.

Let

$$\bar{x} = \sum_{i=1}^N x_i$$

denote aggregate investment.

Investment affects the economy through two channels: (i) the aggregate protection channel, and (ii) the relative allocation channel. The aggregate protection channel expands the total surplus available in the economy, while the relative allocation channel affects how that surplus is distributed across firms.

We therefore distinguish between the protection technology $T(\bar{x})$ and the relative allocation rule $\sigma_i(x; \theta)$, where $\theta \geq 0$ measures the intensity of comparative screening. Firm i 's effective transaction volume is given by

$$q_i = T(\bar{x}) \sigma_i(x; \theta).$$

The interaction of these two channels is the central object of the analysis.

2.2 Aggregate Protection

Let $T: [0, \infty) \rightarrow (0, \infty)$ denote aggregate protection. I impose the following standard conditions.

Assumption 1 (Defense Technology)

1. $T'(x) > 0$.
2. $T''(x) \leq 0$.

Thus investment is socially productive, but with diminishing marginal returns. This channel captures genuine protective value against exogenous threats and is independent of relative ranking.

2.3 Relative Allocation

Let $\sigma_i(x; \theta)$ denote firm i 's allocation share. The allocation rule is assumed to satisfy the following properties.

Assumption 2 (Relative Allocation Rule)

1. Symmetry

If $x_i = x_j$ for all i, j , then $\sigma_i = 1/N$ for all i .

2. Monotonicity

$$\frac{\partial \sigma_i(x; \theta)}{\partial x_i} > 0$$

3. Adding-up

$$\sum_{i=1}^N \sigma_i(x; \theta) = \mathbf{1}$$

4. Amplification

At a symmetric profile $x_i = \dots = x_N = x$, there exists a positive function $H(x)$ such that

$$\frac{\partial \sigma_i(x; \theta)}{\partial x_i} = \theta H(x).$$

The parameter θ governs how sharply marginal differences in investment are translated into differences in relative allocation.

Canonical contest success functions are nested as special cases of this representation (Tullock, 1980; Skaperdas, 1996).

Firm i 's effective transaction volume is therefore given by

$$q_i = T(\bar{x})\sigma_i(x; \theta).$$

2.4 Revenue, Costs, and Welfare

Firm i 's payoff is

$$\pi_i(x) = R(q_i) - C(x_i),$$

where $R: R_+ \rightarrow R$ is increasing and weakly concave, while $C: [0, \infty)$ is increasing and strictly convex.

Assumption 3 (Revenue)

1. $R'(q) > 0$.
2. $R''(q) \leq 0$.

Assumption 4 (Cost)

1. $C'(x) > 0$.
2. $C''(x) > 0$.

Social welfare is total surplus net of investment costs:

$$W(x) = \sum_{i=1}^N R(q_i) - \sum_{i=1}^N C(x_i).$$

The crucial distinction is that relative allocation changes the distribution of $T(\bar{x})$ across firms without changing its aggregate level. Welfare therefore depends on the aggregate protection channel, whereas private incentives depend on both channels.

2.5 Symmetric Equilibrium and Social Optimum

We restrict attention to symmetric allocations.

Definition 1 (Symmetric Nash Equilibrium)

A symmetric Nash equilibrium is an investment level x^{NE} such that (x^{NE}, \dots, x^{NE}) is a Nash equilibrium.

Definition 2 (Symmetric Social Optimum)

The symmetric social optimum x^* solves

$$\max_{x \geq 0} W(x, \dots, x).$$

In the symmetric benchmark, the model sharply isolates the wedge between private and social marginal incentives.

3. Equilibrium Analysis

3.1 Private and Social First-Order Conditions

At a symmetric profile, let x denote aggregate investment, so that each firm chooses x/N , $\sigma_i = 1/N$, and $q_i = T(x)/N$.

Differentiating firm i 's payoff with respect to its own investment yields the symmetric first-order condition for a Nash equilibrium:

$$C' \left(\frac{x^{NE}}{N} \right) = R' \left(\frac{T(x^{NE})}{N} \right) \left[\frac{T'(x^{NE})}{N^2} + \theta H(x^{NE}) T(x^{NE}) \right].$$

The corresponding social optimum internalizes only the effect of x on aggregate protection:

$$C' \left(\frac{x^*}{N} \right) = R' \left(\frac{T(x^*)}{N} \right) \frac{T'(x^*)}{N}.$$

The difference between these conditions is not the presence or absence of protection. Both private and social incentives value the aggregate defense effect. The difference is the positional component generated by relative allocation.

To obtain a transparent closed-form threshold in the baseline symmetric case, normalize to

$$H(x) = \frac{N-1}{N^2},$$

so the private first-order condition becomes

$$C' \left(\frac{x^{NE}}{N} \right) = R' \left(\frac{T(x^{NE})}{N} \right) \left[\frac{T'(x^{NE})}{N^2} + \frac{N-1}{N^2} \theta T(x^{NE}) \right].$$

3.2 The Positional Wedge

Define the marginal difference between private and social incentives at a symmetric profile x as

$$\Delta(x; \theta) = R' \left(\frac{T(x)}{N} \right) \frac{N-1}{N^2} [\theta T(x) - T'(x)].$$

The sign of $\Delta(x; \theta)$ is the sign of $\theta T(x) - T'(x)$. Thus, at any symmetric profile, the private-social wedge is positive when comparative screening is sufficiently strong relative to marginal defense

effectiveness. Evaluated at the symmetric social optimum x^* , this condition yields a sharp criterion for overinvestment in the symmetric benchmark.

The following theorem summarizes the main result.

Theorem 1 (Conditional Overinvestment in the Symmetric Benchmark)

Suppose Assumptions 1–4 hold, an interior symmetric social optimum x^* exists, and a symmetric Nash equilibrium x^{NE} exists. Then:

1. If

$$\theta > \frac{T'(x^*)}{T(x^*)},$$

the private marginal incentive at x^* strictly exceeds the social marginal incentive at x^* , and hence

$$x^{NE} > x^*.$$

2. If

$$\theta < \frac{T'(x^*)}{T(x^*)},$$

the positional component is not large enough to overturn the aggregate defense motive at x^* ; in particular, the private first-order condition is not shifted upward relative to the social optimum at that point, so overinvestment does not arise there.

Proof sketch.

At x^* , the planner's first-order condition equates marginal cost to the marginal aggregate-protection benefit. The private decision maker, by contrast, also internalizes the marginal gain from improving relative allocation. The difference between the private and social marginal conditions at x^* is

$$\Delta(x^*; \theta) = R' \left(\frac{T(x^*)}{N} \right) \frac{N-1}{N^2} [\theta T(x^*) - T'(x^*)].$$

Hence the sign of the wedge is determined by $\theta T(x^*) - T'(x^*)$. When this expression is positive, the private marginal return at x^* lies above the social marginal return. Since C is strictly convex and R is weakly concave, the private first-order condition is satisfied only at a strictly higher investment level, implying $x^{NE} > x^*$.

Figure 2 illustrates the result. The private marginal benefit curve lies above the social marginal benefit curve by exactly the positional wedge. The decentralized equilibrium therefore lies to the right of the social optimum whenever comparative screening is sufficiently strong.

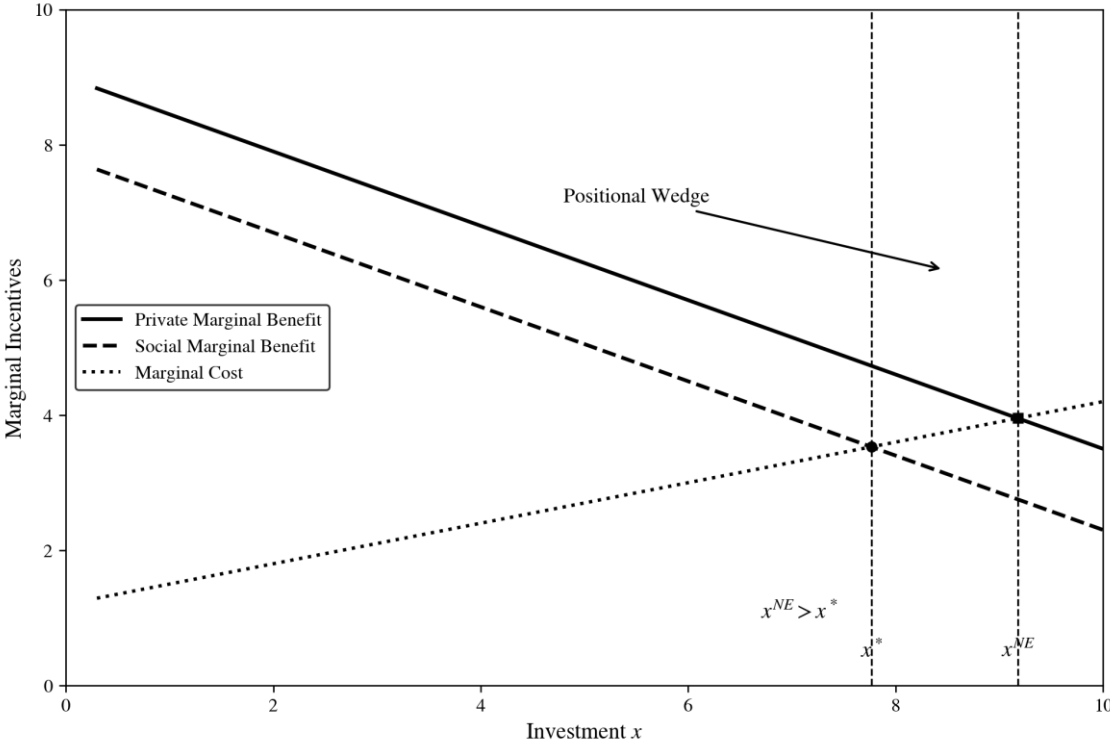


Figure 2. Private versus Social Marginal Incentives

3.3 Interpretation

The theorem shows that overinvestment does not arise because defense is unproductive. Aggregate defense remains socially beneficial. The distortion originates in the coexistence of a productive public-defense channel and a zero-sum relative allocation channel. The equilibrium may therefore display a form of protection-compatible escalation: firms invest heavily in something socially useful, yet the privately chosen level remains inefficiently high.

This distinguishes the model from a pure rent-seeking or wasteful-signaling environment. It also helps explain why excessive security, compliance, or certification effort can arise even when those activities have genuine social value.

4. Transparency as an Amplification Parameter

The previous section treats θ as the intensity of comparative screening. A natural interpretation is transparency. Greater transparency makes marginal differences in investment more visible and therefore more consequential for relative evaluation.

Let θ index the observability or salience of investment. For fixed T and C , higher θ leaves the aggregate protection channel unchanged but increases the positional component of private returns. The equilibrium condition therefore shifts with transparency even though the social return to aggregate protection is unaffected.

Proposition 1 (Transparency Amplification)

Holding T , R , and C fixed, an increase in transparency θ raises the positional component of private incentives without altering the aggregate protection channel. Consequently, whenever the economy is initially below the overinvestment threshold, a sufficient increase in θ can move the economy into the overinvestment region.

The logic parallels the broader literature on public information, but with a different welfare mechanism. In Morris and Shin (2002), public information changes how agents coordinate. Here, transparency changes how intensively firms compete over relative position. It therefore amplifies positional incentives rather than aggregate protection itself.

A useful implication is that transparency need not be monotone in welfare. Starting from low θ , more transparency can improve investment incentives by mitigating underinvestment. Beyond the critical threshold, however, additional transparency generates excessive escalation. The same institutional change may therefore improve efficiency in one regime and worsen it in another.

5. Public Standards and Granularity

If transparency amplifies positional distortion, can regulation preserve the aggregate defense channel while reducing the positional one? This section argues yes. The key is not to suppress investment directly, but to modify the mapping from investment to relative evaluation.

Let relative evaluation depend on a transformed signal $g(x)$, where g is increasing and differentiable. The allocation rule now takes the form $\sigma_i(g(x); \theta)$. The derivative with respect to investment becomes proportional to $\theta g'(x)H(x)$. Thus the effective positional sensitivity is scaled by $g'(x)$.

When public standards, standardized reporting, or coarser audit categories reduce the granularity with which marginal investment differences are translated into ranking, they lower $g'(x)$. The private first-order condition becomes

$$C' \left(\frac{x^{NE}}{N} \right) = R' \left(\frac{T(x^{NE})}{N} \right) \left[\frac{T'(x^{NE})}{N^2} + \frac{N-1}{N^2} \theta g'(x^{NE}) T(x^{NE}) \right].$$

This yields the following implication.

Proposition 2 (Granularity and Mitigation)

Suppose regulation changes the evaluation mapping from x to $g(x)$ with $0 < g'(x) < 1$ over the relevant range. Then the critical threshold for overinvestment becomes

$$\theta > \frac{T'(x^*)}{g'(x^*)T(x^*)}.$$

A reduction in granularity therefore shifts the threshold rightward and attenuates positional escalation while preserving aggregate protection.

The mechanism is structural: public standards improve welfare not by suppressing useful investment, but by separating the aggregate-defense layer from the ranking layer and weakening the translation from effort into rank.

This result is closely related in spirit to the multitask literature (Holmström and Milgrom, 1991). When evaluators place excessive weight on dimensions that are easy to observe and rank, effort is distorted toward those dimensions. In the present setting, coarse evaluation works as an institutional counterweight to excessive fine-grained screening.

6. Welfare Decomposition

6.1 Aggregate Defense versus Relative Escalation

The main welfare insight can be made explicit by separating equilibrium investment into two conceptual components:

$$x^{NE} = x^* + \text{Positional Escalation.}$$

At the social optimum, firms invest up to the point where the marginal aggregate protection effect equals marginal cost. Beyond that level, additional investment is privately attractive only because it improves relative standing. Since the relative term redistributes $T(x)$ rather than expanding it, the excess part of equilibrium investment is pure positional escalation.

6.2 A Useful Decomposition

At a symmetric profile, the difference between private and social marginal returns is

$$\Delta(x; \theta) = R' \left(\frac{T(x)}{N} \right) \frac{N-1}{N^2} [\theta T(x) - T'(x)].$$

This expression highlights two sufficient statistics:

- (i) Screening intensity θ , which scales the positional component;
- (ii) The ratio $T'(x)/T(x)$, which summarizes how much additional aggregate protection is generated relative to the already protected base.

When θ rises or $T'(x)/T(x)$ falls, positional distortion becomes more likely. The welfare problem is therefore not simply “too much investment”. It is too much investment in the ranking dimension of a socially useful activity.

6.3 Why the Model is not about Wasteful Protection

The model does not imply that security or compliance investment is intrinsically wasteful. On the contrary, $T'(x) > 0$ ensures genuine social value. The distortion appears because relative screening converts part of that valuable activity into a status race. This is why the paper’s central mechanism is better understood as protection-compatible escalation than as ordinary rent-seeking.

The distinction matters for policy. If the investment itself were socially worthless, the prescription would be straightforward suppression. Here that would be a mistake. Since aggregate protection is valuable, policy should instead target the translation from effort into relative rank.

7. Conclusion

This paper develops a structural theory of investment under dual incentive structures. When investment both enlarges aggregate protection and affects relative allocation, private and social marginal incentives diverge systematically. The key distortion identified here is not that investment lacks social value. On the contrary, aggregate defense remains productive. The inefficiency arises because relative allocation adds a positional return to privately chosen investment without increasing aggregate surplus.

The main result is a threshold characterization of this wedge in the symmetric benchmark.

Overinvestment arises when comparative screening intensity becomes sufficiently strong relative to the marginal effectiveness of aggregate protection. Transparency amplifies this positional component, whereas public standards mitigate it by reducing the granularity with which effort is translated into relative rank.

The broader implication is structural. In environments where socially useful investment is filtered through comparative evaluation, efficiency depends not only on the level of investment, but on the institutional mapping from effort into rank. The relevant policy margin is therefore not simple suppression, but the design of evaluation and screening rules.

Investment can be socially productive and yet excessive in equilibrium. That possibility is the central message of the paper.

Appendix

Appendix A. Robustness to Labor Adjustment Frictions

In practice, investment often entails not only direct technological cost but also labor adjustment costs, administrative burdens, and organizational frictions. I show that such frictions change the level of investment but not the source of distortion.

Let total cost be

$$\tilde{C}(x) = C(x) + L(x),$$

where $L'(x) \geq 0$ and $L''(x) \geq 0$. Then total cost remains strictly convex.

The symmetric equilibrium condition becomes

$$\tilde{C}'\left(\frac{x^{NE}}{N}\right) = R'\left(\frac{T(x^{NE})}{N}\right) \left[\frac{T'(x^{NE})}{N^2} + \frac{N-1}{N^2} \theta T(x^{NE}) \right],$$

while the social optimum satisfies

$$\tilde{C}'\left(\frac{x^*}{N}\right) = R'\left(\frac{T(x^*)}{N}\right) \frac{T'(x^*)}{N}.$$

The wedge term remains

$$\Delta(x; \theta) = R'\left(\frac{T(x)}{N}\right) \frac{N-1}{N^2} [\theta T(x) - T'(x)].$$

Hence the threshold condition for overinvestment is unchanged:

$$\theta > \frac{T'(x^*)}{T(x^*)}.$$

Labor adjustment frictions therefore reduce the overall scale of investment but do not eliminate the structural asymmetry between private and social incentives. The inefficiency continues to arise from the positional component embedded in relative screening.

Appendix B. Regime Interpretation and Shock Amplification

This appendix interprets the static threshold condition under changing environments. The purpose is not to introduce a fully dynamic model, but to clarify how the sufficient statistics in the main theorem organize regime shifts.

Let the economy face a state $s \in S$ summarizing technological turbulence, threat intensity, or regulatory scrutiny. Allow both aggregate defense effectiveness and screening intensity to depend on s :

$$T(\bar{x}; s), \quad \theta(s).$$

The overinvestment condition in state s becomes

$$\theta(s) > \frac{T_{\bar{x}}'(\bar{x}^*(s); s)}{T(\bar{x}^*(s); s)}.$$

A regime shift occurs when a change in s increases $\theta(s)$, decreases T_x'/T , or both. Two comparative statics are especially important:

1. Screening amplification: $\theta(s)$ increases.
2. Defense weakening: the ratio $T_x'(\bar{x}^*(s); s)/T(\bar{x}^*(s); s)$ decreases.

Either movement makes positional distortion more likely. If both occur simultaneously, the economy may cross abruptly from underinvestment to overinvestment.

This yields a natural interpretation of a window of vulnerability. When screening intensity rises faster than the marginal effectiveness of aggregate defense, total investment can increase even while the incremental contribution of investment to systemic resilience declines. In such a regime, escalation coexists with fragility.

Public standards shift the boundary again. If standardized evaluation reduces effective granularity, the threshold becomes

$$\theta(s) > \frac{T_x'(\bar{x}^*(s); s)}{T(\bar{x}^*(s); s)} \cdot \frac{1}{g'(x^*(s))}.$$

Lower $g'(x^*(s))$ shifts the threshold rightward and shrinks the region in which escalation occurs.

Figure B1 illustrates this regime interpretation.

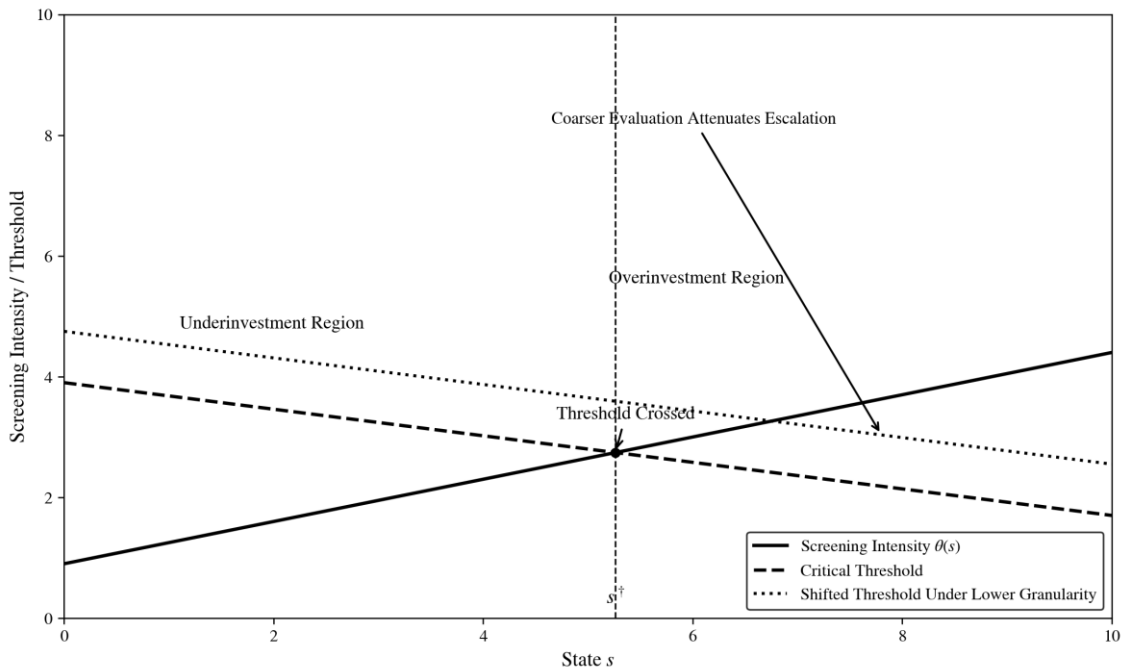


Figure B1. Threshold Crossing and Regime Shift

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