Graph Signal Processing Summer School

Proposal for a research school at CIRM during the first semester 2018

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Proposal for a Summer School on Graph Signal Processing to be held in 2018 at CIRM in Marseille, France.

1 Summary

The proposed Summer School aims to provide PhD students and researchers with insights on advanced tools and methodologies in Graph Signal Processing (GSP) research. The goal is to disseminate state-of-the-art research about signal processing on graphs as well as discussing and defining new directions both on the theoretical developments of the GSP framework and on its applications to real-world science and engineering problems. There will be a set of introductory level lectures for those interested to apply GSP tools to their research or scientific problems as well as a set of advanced topics geared towards researchers in the field. The exact topics have yet to be discussed with potential speakers. They will offer a mix of theoretical and applied talks.

Summer school participants will have the opportunity to learn and study innovative algorithms and systems in the signal processing field in an interactive and stimulating way. Each day, after the lectures, the speakers will be available for face-to-face discussion with participants who are interested in strengthen their knowledge of the covered material. Students will have the possibility to present their work to let other participants and the lecturers know about their research activity. There will be free time for students to discuss, exchange ideas, brainstorm or engage in debates.

The first edition of this school, previously named *Key Insights on Networks and Graphs*[1], was held in Leukerbad, Switzerland, in June 2015.

2 Résumé

L'école d'été proposée a pour objectif de donner aux doctorants un aperçu des outils et méthodologies en Traitement du Signal sur Graphe (TSG). Le but est de disséminer l'état de l'art en TSG ainsi que de discuter et définir de nouvelles directions tant sur le développement théorique des fondements du TSG que sur ses applications aux problèmes scientifiques et d'ingénieries. Il y aura un ensemble de cours introductifs pour ceux qui sont intéressés à l'application des outils du TSG à leurs recherches ou à leurs problèmes ainsi qu'un ensemble de cours avancés orientés vers les chercheurs du domaine. Les thèmes

[1] http://lts2.epfl.ch/summerschool

abordés doivent encore être discutés avec les conférenciers potentiels. Ils proposeront un mélange de sujets théoriques et appliqués.

Les participants à cette école d'été auront l'opportunité d'apprendre et d'étudier des algorithmes et systèmes innovants en traitement du signal de manière interactive et stimulante. Chaque jour, après les cours, les conférenciers seront disponibles pour discuter avec les participants intéressés à consolider leurs connaissances de la matière abordée. Les doctorants auront la possibilité de présenter leur travail afin que tant leurs collègues que les conférenciers soient au courant de leurs activités de recherche. Du temps libre sera aménagé pour que les doctorants puissent discuter, échanger des idées ou s'engager dans des débats.

La première édition de cette école, précédemment nommée *Key Insights on Networks and Graphs*[2], a eu lieu à Loèche-les-Bains, en Suisse, en juin 2015.

3 Scientific content

With the advent of Big Data, an ever increasing amount of raw data is becoming available and advanced models are needed to extract meaningful information from data and identify relational structures that facilitate different data analysis tasks such as recognition or inference. Graph Signal Processing (GSP) enables the analysis and processing of signals that reside on irregular (non-Euclidean) domains, such as infrastructure or social networks. Graphs may further be used to incorporate external information into data models, such as hyperlinks or references between documents, and to capture the underlying geometrically complex manifold structure of the data.

Graphs and Signal Processing. Graphs are generic data representation forms that are useful for describing the geometric structures of data domains in numerous applications, including social, energy, transportation, sensor, and neuronal networks. The weight associated with each edge in the graph often represents the similarity between the two vertices it connects. The connectivities and edge weights are either dictated by the physics of the problem at hand or inferred from the data. For instance, the edge weight may be inversely proportional to the physical distance between nodes in some network. The data on these graphs can be visualized as a finite collection of samples, with one sample at each vertex in the graph. Collectively, we refer to these samples as a graph signal.

The purpose of GSP is to exploit the underlying structure to analyze, process and learn signals residing on graphs. Classical GSP tasks include, among others, filtering, denoising, inpainting and compressing. An important consideration when designing such tools is their computational complexity, which can often be improved by spectral relaxations or numerical approximations. Efficiency is key to cope with the ever growing size of datasets.

Graphs as data models. There exists two paradigms to model the underlying structure of datasets using graphs: 1) constructing a *feature graph*, where nodes represent features and samples are graph signals, or 2) a *data graph*, where nodes represent samples and graph signals are functions of the data. We then identify two regimes: 1) when the graph structure is given by the application, and 2) when the graph has to be constructed from features or data. Below we elaborate on three of those four combinations.

Examples of applications where a feature graph structure is given are found in many different engineering and science fields. In transportation networks, we may be interested

[2] http://lts2.epfl.ch/summerschool

in analyzing epidemiological data describing the spread of diseases, census data describing human migration patterns, or logistics data describing inventories of trade goods. Other examples include gene expression patterns defined on top of gene networks, the congestion level at the nodes of a telecommunication network, weather data measured by a network of sensors and patterns of brain activity defined on top of a brain network.

In statistical learning problems, GSP is a meaningful paradigm even when the graph structure is not given by the problem but might be inferred from data. In such cases, the graph is a discrete approximation of a manifold embedded in an Euclidean space: each node represents a sample and the weights represent similarities between samples. Signals on those graphs may be the output of a classification or regression model which we could constrain to be smooth or piece-wise constant on the data graph. Applications include machine vision (e.g. semi-local graphs between pixels) and automatic text classification. Graph-based methods are especially popular for the (transductive or inductive) semi-supervised learning problem where the objective is to classify unknown data with the help of a few labeled samples and a large pool of unlabeled training samples.

As an example of intrinsic data graph, instead of defining weights as similarities between vector representations of documents (from e.g. the bag-of-word model), external information might be incorporated by defining binary weights as hyperlinks or references between documents.

Theoretical understanding. Besides particular applications, the summer school will also showcase the advancement of the understanding of network data by redesigning traditional tools originally conceived to study signals defined on regular domains (such as time-varying signals or spatially varying images and fields) and extending them to analyze signals on the more complex graph domain.

The irregular nature of the modeled structures often makes classical signal processing concepts only partially true. The main similarities and differences between classical signal processing and GSP will be presented.

Examples of theoretical topics to be addressed include graph fundamental limits such as sampling theorems and uncertainty principles, convergence results establishing the link between the discrete nature of graphs and the continuous world of manifolds, as well as the theoretical foundations of different graph transforms.

Relevance. The last few years have seen significant progress in the development of theory, tools, and applications of GSP. This school is intended to disseminate ideas to a broader audience and to exchange ideas and experiences on the future path of this emerging field. As the interest grows, a summer school[3] has been organized in 2015 in Switzerland and a workshop[4] will take place this year in the US.

Last year, Pascal Frossard got invited to give a plenary talk in the Sampling Theory and Applications (SampTA) conference[5] as well as an overview talk about multimedia graph-based processing at the Conference on Multimedia and Expo (ICME)[6], which shows the interest of the multimedia community in GSP. While mainly focusing on complex and dynamical networks, the *Network Science Thematic Semester*[7], which comprises two workshops and a conference that will take place in May and June 2016 in Lyon and Marseille, in

[3] http://lts2.epfl.ch/summerschool
[4] http://alliance.seas.upenn.edu/~gsp16/wiki
[5] http://www.american.edu/cas/mathstat/sampta2015
[6] http://www.icme2015.ieee-icme.org
[7] http://project.inria.fr/netspringlyon

France, features some keynotes and presentations from the GSP community, notably with the interventions of Pierre Vandergheynst, José M.F. Moura and Alejandro Ribeiro. The IEEE Image, Video, and Multidimensional Signal Processing (IVMSP) workshop 2016[8], focused on image and video processing, mentions GSP and manifold modeling in its call for papers and Pierre Vandergheynst will give a plenary talk on signal and image processing on graphs.

Selected topics. Please refer to published review papers [46, 42, 10, 17] for a broad technical overview as well as a discussion of the challenges in GSP. Below is a selection of applicable topics with selected (recent) references, which we hope help to demonstrate the increasing popularity of GSP. The selection of topics to be presented will be discussed with the speakers and will depend on their interests.

- Theoretical foundations and convergence results [3, 53]
- Graph spectral theory [9, 3, 12, 43, 34, 8, 14]
- Sampling and recovery of graph signals [41, 52, 2]
- Graph filter and filter bank design [21, 36, 16, 28, 29, 13]
- Uncertainty principles and other fundamental limits [40, 1, 38]
- Graph coarsening, multi-scale graph signal processing [48, 11, 31, 27, 25]
- Graph signal transforms [49, 20, 32, 10, 27, 35]
- Graph filter identification [51]
- Graph construction and graph topology inference [26, 15]
- Learning graph embeddings [53, 57, 4]
- Statistical graph signal processing [39, 54, 19]
- Graph-based image and video processing [18, 30, 45]
- Graph-based 3D multimedia processing [55, 50, 37, 44]
- Machine learning with graph regularization [47, 5, 58, 56, 6]
- Applications to neuroscience and other medical fields [23, 22]
- Applications to economics and social networks
- Applications to infrastructure networks such as communication, transportation, power networks [33, 24, 7]

4 Speakers

We plan to invite five speakers to give lectures. We expect one of them, the first to present on Monday, to introduce the field and give a motivational talk. The purpose of this overview talk is to help novice attendants to follow the more advanced talks. Speakers are selected such that all of those three main tracks and possibly a fourth side track are covered:

1. Track 1: Introduction to GSP
2. Track 2: Foundations of GSP
3. Track 3: Recent innovation and applications of GSP
4. Side track: Optimization

As there is a strong optimization community in France which could easily travel to CIRM for a talk, we envision to introduce a side track about optimization, which is quite relevant to GSP. We'll discuss with those speakers on how to adapt their talks to our audience.

[8] http://www.ivmsp2016.org

We are very enthusiastic about the impact this exchange could have. We however won't contact those speakers yet, as it may not happen should this proposal be rejected (because the event may then be located in another place with different local opportunities).

Below is the list of speakers we contacted to lecture at this school. As we don't know the date and place of the school, we didn't ask them to confirm their attendance yet.

- Track 1
  - Pierre Vandergheynst, EPFL, Switzerland
- Track 2
  - Ulrike von Luxburg, University of Tübingen, Germany
  - Daniel Spielman, Yale University, USA
- Track 3
  - Jure Leskovec, Stanford University, USA
  - Rémi Gribonval, INRIA Rennes, France

In case the aforementioned speakers cannot come, below is another list (in no particular order) of potential replacements. The diversity of this school is again reflected in the broad range of considered speakers. Given this large list, we are pretty confident that enough of them will be able to attend our event, given how much time is left to arrange their attendance.

- Track 1
  - José M. F. Moura, Carnegie Mellon University, USA
- Track 2
  - Mikhail Belkin, Ohio State University, USA
  - Fan Chung Graham, UC San Diego, USA
- Track 3
  - Antonio Ortega, University of Southern California, USA
  - Mauro Maggioni, Duke University, USA
  - Abderrahim Elmoataz, University of Caen Basse Normandie, France
  - Risi Kondor, University of Chicago, USA
  - Michael Rabbat, McGill University, Canada
  - Pierre Borgnat, ENS Lyon / CNRS, France
  - Patrick Flandrin, ENS Lyon / CNRS, France
  - Sergio Barbarossa, Sapienza University of Rome, Italy
  - Philip A. Chou, Microsoft Research, USA
  - Naoky Saito, UC Davis, USA
  - Alejandro Ribeiro, University of Pennsylvania, USA
  - Ronald Coifman, Yale University, USA
  - Pier Luigi Dragotti, Imperial College London, Great Britain
  - Jelena Kovačević, Carnegie Mellon University, USA
  - Pascal Frossard, EPFL, Switzerland
  - Alain Barrat, Université Aix-Marseille / CNRS, France
  - Jon Kleinberg, Cornell University, USA
- Side track
  - Francis Bach, INRIA Paris, France
  - Patrick Louis Combettes, Université Pierre et Marie Curie, France

– Jean-Christophe Pesquet, Université Paris-Est, France
– Jalal Fadili, ENSI Caen, France
– Gabriel Peyré, Université Paris-Dauphine / CNRS, France

5 Participants

This summer school targets an international audience, with a diverse set of speakers and potentially interested research labs. It will gather people using similar tools and techniques in diverse fields such as signal processing, graph analysis and machine learning. If the proposal is accepted, we will activate our networks and notify all the interested parties we know about. In addition to the laboratories of the potential speakers, we expect the students of the following groups to be interested, some of which we have collaborated with in the past.

- Kannan Ramchandran, UC Berkeley, USA
- Vincent Gripon, Télécom Bretagne, France
- David Shuman, Macalester College, USA
- Michael M. Bronstein, USI, Switzerland
- Camille Roth, Humboldt Universität / CNRS, Germany
- Renaud Lambiotte, University of Namur, Belgium
- Alfred Hero, University of Michigan, USA
- Vittoria Colizza, Inserm, France
- Patrick Thiran, EPFL, Switzerland
- Matthias Grossglauser, EPFL, Switzerland

We however do not expect more than 40 participants, (the last edition attracted 32 students), such that we propose to co-locate it with the *Dynamique des Systèmes Biologiques* summer school, whose organizers expect around 40 participants too. As such we chose the same dates and will limit the number of attendees to 40. That way, all participants from the two schools can be accommodated at CIRM. The lectures from the two events will take place in two different auditoriums, namely at CIRM and CPPM[9]. While the scientific part will be organized independently, we plan to jointly organize the social events (lunch, dinner, excursion) during which students may exchange and eventually develop future collaborations.

6 Others

6.1 Organization

This school is to be organized by:

- Michaël Defferrard, PhD student, EPFL, Switzerland
- Nathanaël Perraudin, PhD student, EPFL, Switzerland
- Yann Schoenenberger, PhD student, EPFL, Switzerland
- Dorina Thanou, scientist, EPFL, Switzerland

and the scientific committee is composed of:

- Pascal Frossard, associate professor, EPFL, Switzerland
- Bruno Torrésani, full professor, Aix-Marseille University, France

[9] Confirmed by Bruno Torrésani.

- Mauro Maggioni, full professor, Duke University, USA

We want to thank Yannick Boursier, Lionel Martin, Johan Paratte, Benjamin Ricaud and Xavier Bresson who helped us to prepare this proposal and will continue to provide support should it be retained.

6.2 Gender parity analysis

As in many other engineering fields, there is an unfortunate imbalance between the number of men and women in the GSP community. To the best of our knowledge, Fan Chung Graham, Ulrike von Luxburg and Jelena Kovačević are the only women professors in the field. All three rank high in our priority list and we would be honored to have such distinguished speakers at the summer school.

As for the participants we expect to be representative of the field. At EPFL the gender distribution is 10 women and 25 men in the labs of Prof. Frossard[10] and Prof. Vandergheynst[11].

6.3 ECTS credits

We suggest to grant 2 ECTS points to Ph.D. students who participate in the school. To that end, a certificate of participation will be delivered at the end of the week; it is then up to their home institutions to choose whether this suggestion should be followed.

6.4 Location

With the idea that nice places can help open the mind, we would be delighted to host this event in the beautiful coastal area of Marseille. It would further be in contrast to the previous edition which was hosted in Leukerbad, a small Swiss town up in a valley in the Alps. As Leukerbad, Marseille is a place which is quiet for the mind and relaxing for the body. In addition to its intrinsic attractiveness as a touristic city, Marseille is easily accessible by plane, which is important for our international audience. Its closeness to the French Riviera and Paris makes it an even more attractive place for overseas speakers and participants.

6.5 Schedule

The school will last five days, from Monday morning to Friday afternoon. Rooms will be available from Sunday evening to Friday morning. Participants will have to arrive and install themselves at CIRM on Sunday as lectures will start Monday morning. We plan to finish early on Friday and to offer an excursion on Wednesday afternoon, which leaves us with four days for lectures. Following the schedule shown in Table 1, we have room for 24 lessons of 45 minutes. Each of the five main track speakers will be allocated four lessons and each side track speaker will be allocated one lesson. Exact allocations will be discussed with the speakers.

Attendees will be offered the opportunity to present their work. Interested students will have to provide an abstract. The scientific committee will choose the most relevant ones, which will be allocated a 20 to 30 minutes time slot at the end of a day. The participation to these presentations is optional. Additionally, we schedule some free time after lunch

[10] http://lts4.epfl.ch
[11] http://lts2.epfl.ch

to discuss and exchange ideas about future developments of the field. Each day, we may propose a different topic for students to brainstorm and discuss.

Depending on funding opportunities and the enthusiasm of the community, notably at GSP2016[12], this event may grow in size (in terms of speakers and attendants) and become a mix of long tutorial lectures and short lectures. In that case, student presentations may be morphed into a poster session, but we definitely want students to be able to present their work.

8:00	-	8:45	Breakfast
9:00	-	9:45	Lesson 1
10:00	-	10:45	Lesson 2
10:45	-	11:15	Coffee break
11:15	-	12:00	Lesson 3
12:15	-	13:15	Lunch
13:30	-	14:45	Free time, discussions
15:00	-	15:45	Lesson 4
16:00	-	16:45	Lesson 5
16:45	-	17:15	Coffee break
17:15	-	18:00	Lesson 6
18:30	-	19:30	Student presentations
20:00			Dinner and social events

Table 1: Schedule of a typical day.

6.6 Dates

As we aim to co-locate with another summer school, we jointly propose the following dates (it may be in a different preference order):

1. June 5th
2. May 15th
3. April 23rd

6.7 Budget

The CIRM subsidizes meals and accommodations (in two beds rooms) for 40 participants. As we plan to co-locate with another school, the CIRM would provide half of his funding to us while the other half will go to the other event. This will thus cover 20 of our participants. The CIRM package is offered at 88.5€ per day, which accounts for $88.5 \cdot 5 = 442.50$€ per participant.

Invited speakers will be provided with meals and an individual room at CIRM. A double room or a flat rate will be offered to those who prefer to self-organize, e.g. for family reasons. The CIRM prices are: 61.80€ per night for a single room including breakfast, 15.45€ per meal. We thus budget $61.80 \cdot 5 + 15.45 \cdot 10 = 465.5$€ per speaker. We plan to defray their travel expenses. While the exact amount will depend on the available funding, we budget 200€ for EU speakers and 700€ for US speakers

If this proposal is accepted, we will request additional funding to local communities in

[12] Workshop to be held in May 2016 in Pennsylvania, USA.

the CIRM area[13] and to the Seasonal Schools in Signal Processing (S3P) Program[14] of the IEEE Signal Processing Society (SPS). While there is no call for proposals for 2018 yet, the winter 2016-2017 version states that "an SPS contribution of up to 5k US dollars can be included in the budget". Additional funding opportunities in Switzerland include the SNF[15], the KTI[16] and the Hasler foundation[17].

To cover additional expenses and to incentivize participants to be "active", we further ask for a registration fee of 200€ per participant from academia and 800€ for non-academic people.

A detailed budget can be found in tables 2 and 3.

Description	Amount (EUR)	Quantity	Expenditures (EUR)
Meals and lodging (participants)	442.50	40	17'700
Meals and lodging (speakers)	465.50	5	2'328
Travel defrayal for EU speakers	200	3	600
Travel defrayal for US speakers	700	2	1'400
Excursion	30	44	1'320
Miscellaneous	502	1	502
Total			**23'850**

Table 2: Budgeted expenditures.

Description	Amount (EUR)	Quantity	Revenues (EUR)
CIRM subsidies	442.50	20	8'850
IEEE SPS contribution	5000	1	5'000
Local sponsoring	2000	1	2'000
Registration fees	200	40	8'000
Total			**23'850**

Table 3: Budgeted revenues.

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